A Second-Best Argument for Low Optimal Tariffs on Intermediate Inputs*

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December 2022

Abstract

We derive a new formula for the optimal uniform tariff in a small-country, heterogeneous-firm model with roundabout production and a nontraded good. Tariffs are applied on imported intermediate inputs. First-best policy requires that markups on domestic intermediate inputs are offset by subsidies. In a second-best setting where such subsidies are not used, roundabout production and the monopoly distortion in the traded sector create strong incentives to lower the optimal tariff on imported inputs. In a quantitative version of our two-sector small open economy, we find that the optimal tariff is lowered under nearly all parameter values considered, and can be negative.

Keywords: trade policy, monopolistic competition, gains from trade, input-output linkages

JEL Codes: F12, F13, F17, F61

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1 Introduction

The use of tariffs to protect traded goods such as manufactures has a long history. In his famous *Report on Manufactures*, Alexander Hamilton argued for moderate tariffs combined with direct subsidies to promote manufacturing. Opposition to these subsidies came from Thomas Jefferson and James Madison, who favored even higher tariffs, and Madison's administration would put in place the first protectionist tariff in the United States (Irwin, 2004). The administration of President Donald Trump enacted tariffs, often at 25%, to protect several manufacturing industries and against a broad range of products from China. Significantly, the Chinese products were initially selected to minimize the direct impact on consumer prices, leaving American businesses facing the brunt of tariffs on their imported inputs (Fajgelbaum, Goldberg, Kennedy and Khandelwal, 2020).

Does modern trade theory offer any new answer to this old question of whether to protect the traded sector? To answer this, we investigate a small open economy (SOE) with two sectors – one traded and the other nontraded – and with heterogeneous firms, monopolistic competition and CES preferences (as in Melitz, 2003). We adopt a Pareto distribution for productivity (as in Chaney, 2008) and also roundabout production.[1] As described in section 2, the differentiated intermediate inputs in each sector are bundled into a finished good that is sold to home consumers and firms in that sector, but not traded, while the differentiated inputs are traded in one sector. A tariff is applied to imports of these differentiated intermediate inputs.

Demidova and Rodríguez-Clare (2009) obtain a formula for the optimal uniform tariff in a SOE with one sector, no roundabout production and heterogeneous firms which we denote by t^{het}. Because there is no roundabout production, we can think of this tariff as applying to imported final good varieties. They argue that this single tariff instrument obtains the first-best by offsetting two distortions: the need to correct for the markup on domestic final goods (by applying a tariff equal to that markup) and the externality present because imported varieties bring surplus that is not taken into account in domestic spending (by slightly lowering the tariff). When there is roundabout production, however, then t^{het} does not correct for the markup on domestic input varieties that is passed-through to the price of the bundled finished good, which is further used as an input to the

[1] Roundabout production means that the output of a sector is used as an input into the same sector: see Krugman and Venables (1995) and Yi (2010).

production of other differentiated inputs. Labor is also used in production, so that markup distorts the use of the finished good relative to labor. In addition, the presence of a second (nontraded) sector creates a further monopoly distortion. The question we address is: what is the optimal tariff on the imported inputs, in the absence of other policy instruments?[2]

In a closed economy, analyzed in section 3, we show that the distortion created by the markup on differentiated inputs is corrected by applying subsidies to the finished good purchased in both sectors.[3] In the open economy analyzed in section 4, first best policy requires subsidies on the finished goods in addition to the tariff t^{het}. When subsidies are not used, however, then the second-best policy in an open economy is to lower the import tariff below t^{het}, thereby lowering the price of the finished good in the traded sector. *Our key result shows conditions under which the optimal second-best tariff on imported varieties is below* t^{het}*, due to the presence of roundabout production and the monopoly distortions in both sectors.*

We obtain the optimal uniform, second-best tariff as a fixed-point of a formula described in section 5 that has two new terms: a term M that reflects the relative monopoly distortion between the traded and nontraded sectors; and a term R that reflects roundabout production in the traded sector as well as the monopoly distortion there. In section 6 we consider a quantitative version of our two-sector SOE, where we find that the optimal tariff is lower that t^{het} under nearly all parameter values considered, and can be negative.[4] Further conclusions are in section 7.

1.1 Related Literature

Costinot, Rodríguez-Clare and Werning (2020) analyze optimal tariffs on final differentiated goods with very general tastes and technologies, and they show that optimal tariffs can be lowered (and

[2] We are assuming that the imported differentiated good is not purchased by consumers directly. If so, and if the government could prevent resale between consumers and firms, then it is possible that a different tariff should be applied on the two groups. But this action would not offset the need to charge a *low* tariff on the input varieties that firms purchase – as we shall argue – so as to offset the markup on the domestic varieties, that is passed-through to the price of the bundled, finished good sold to firms

[3] For consumers, the first-best subsidy is in relative terms (see section 3), since it does not matter if the consumer prices in both sectors are high provided that the tax revenue is redistributed . But for firms purchasing the finished good, the subsidy must *exactly* offset the markup that is passed-through from the input varieties.

[4] In our working paper Caliendo, Feenstra, Romalis and Taylor (CFRT, 2021), we analyze a 186-country, 15-sector quantitative model for 2010 with a general input-output structure. For manufacturing, the one-sector, no roundabout, first-best tariff is 27.3% for our parameter values. We find that the optimal second-best tariff has a median value of only 10% (or 7.5% for countries with above-median shares of manufacturing production), and is negative for five countries: Bhutan, Myanmar, New Caledonia, Hong Kong, and Spain.

even made negative) by having a non-Pareto distribution for productivity or linear foreign preferences. They are the first to extend the analysis to *nonuniform* tariffs, and they find that the importing country should use an import subsidy on the least efficient foreign exporters. Haaland and Venables (2016) demonstrate a potential second-best role for reduced trade taxes to offset a monopoly distortion, as does the earlier work by Flam and Helpman (1987). These papers all focus on trade in final goods, while the impact of tariffs on inputs along global supply chains is examined by Antràs and Chor (2021), Beshkar and Lashkaripour (2020), Blanchard, Bown and Johnson (2016) and Grossman and Helpman (2021).

Recently, Antràs, Fort, Gutiérrez and Tintelnot (2022) have analyzed "tariff escalation," which means higher (optimal) tariffs on final goods than on intermediate inputs. Their model and ours differ in many of the details: they have two sectors with sequential production, with the strongest results obtained when labor is used in the downstream sector; whereas we have two sectors with roundabout production, and no labor used downstream. Despite these differences, we believe that the underlying distortion is the same and arises from the markup on domestic inputs. As a result, subsidizing domestic inputs (in the first-best) or lowering the tariff on imported inputs (in the second-best) is needed to offset those markups.[5]

Our work is most closely related to Lashkaripour and Lugovsky (2020) and Lashkaripour (2021). The former authors analyze optimal uniform first-best tariffs with multiple sectors and input-output linkages. When considering second-best tariffs, however, they do not incorporate these linkages. Still, we build on their result that the first-best policy in the presence of input-output linkages will be to offset the markups charged by sellers of intermediate inputs by providing a subsidy to those buyers (and we show the same result in a closed economy). Our main interest is in the second-best optimal tariff in the absence of the subsidy offsetting the markup.

Lashkaripour (2021) analyzes the second-best use of tariffs in a setting that incorporates input-output linkages. He assumes that there is "duty drawback" on the tariffs applied to imported

[5]There is one important distinction between our models, which arises from the impact of a tariff on intermediate inputs on domestic entry into that sector. Because we have only one traded sector, the import tariff is equivalent to an export tax on that sector (due to Lerner symmetry) and it inhibits entry. In contrast, Antràs, Fort, Gutiérrez and Tintelnot (2022) have two traded sectors, so that a tariff on the upstream sector alone is not equivalent to an export tariff on that sector, and it is quite possible that entry increases as in the *firm-delocation* literature. See further discussion in sections 2.1 and 7.

intermediate inputs, meaning that those duties are forgiven when the imported inputs are used in the production of exported goods. We do not rely on this assumption. Despite differences in the questions that we address (Lashkaripour analyzes Nash-equilibrium tariffs whereas we investigate tariffs for a SOE), there are similarities in our results. Lashkaripour stresses that the welfare impact of tariffs depend on their ability to raise wages in the importing country, and that input-output linkages reduce the calculated optimal tariffs. We similarly show that the wage impact of tariffs is reduced due to roundabout production in the traded sector, which is one reason for the optimal tariff to be lowered. In the presence of markups, Lashkaripour (2021) argues that the second-best tariff should offset those domestic distortions. We likewise find that the monopoly distortion in the traded sector – in conjunction with roundabout production there – is another reason to lower the optimal tariff. Adding the nontraded sector creates a further distortion, and whether this increases or decreases the tariff depends on which sector is more distorted.

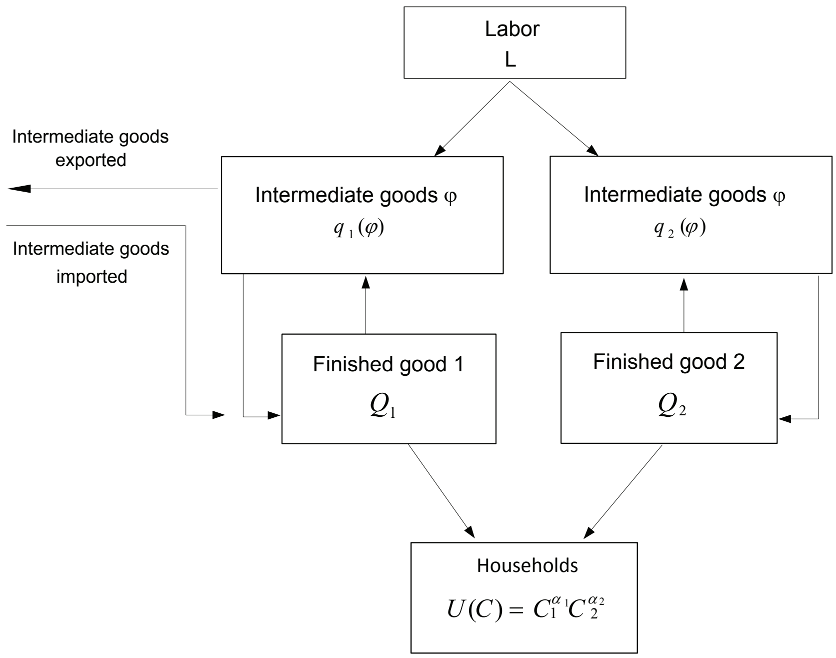
2 Two-Sector Economy with Roundabout Production

We analyze a two-sector Melitz (2003)-Chaney (2008) model with roundabout production, similar to Arkolakis, Costinot, and Rodríguez-Clare (2012, section IV) and Costinot and Rodríguez-Clare (2014). We summarize key equations here and Online Appendix A describes the full model with heterogeneous firms, while Appendix B outlines the model with homogeneous firms.

There are two countries, where the home country is a small open economy (SOE) and the foreign country is denoted by an asterisk. As illustrated in Figure 1, there are two sectors $s = 1, 2$ at home, where sector 1 is traded and sector 2 is nontraded. In both sectors, firms produce differentiated inputs under monopolistic competition, which are costlessly bundled into a *finished good* in CES fashion, with elasticity $\sigma_s > 1$. The finished good is nontraded in both sectors, and it is sold to domestic consumers as a final good and also to domestic firms in the same sector as an intermediate input, used to produce differentiated inputs (e.g., firms produce machinery parts using machines). In sector 1, the imported differentiated inputs are subject to iceberg costs and a tariff, where one plus the *ad valorem* tariff is denoted by t_1.

The finished output in each sector has quantity Q_s, price index P_s, and value $Y_s \equiv P_s Q_s$.

Figure 1: Schematic production structure



With roundabout production, the marginal cost of producing a differentiated input for a firm with productivity $\varphi_s = 1$ in sector s is

$$c_s \equiv w^{(1-\gamma_s)} P_s^{\gamma_s}, \tag{1}$$

where $0 < (1-\gamma_s) \leq 1$ is the labor share so that γ_s indicates the amount of roundabout production. We refer to (1) as the *input cost index*.

A mass of firms N_s^e incur fixed labor costs of entry f_s^e to enter in each sector. In both the homogeneous and heterogeneous firms models, that mass is endogenously determined from the full-employment conditions for the economy. With homogeneous firms, all firms receive a productivity of unity; with heterogeneous firms firms receive a productivity draw from a Pareto distribution, $G_s(\varphi_s) = 1 - \varphi_s^{-\theta_s}$, with $\varphi_s \geq 1$ and $\theta_s > \sigma_s - 1$. As is familiar in the Melitz-Chaney model, firms choose to produce the differentiated input for the domestic market or to export if their productivities exceed some *cutoff* levels, and in each case, the firms then incur additional fixed labor costs.

Consumers have Cobb-Douglas preferences over final goods in the two sectors, with

$$U = C_1^{\alpha_1} C_2^{\alpha_2}, \quad \alpha_1 + \alpha_2 = 1, \quad \alpha_1 > 0 \text{ and } \alpha_2 \geq 0, \tag{2}$$

where α_s is the expenditure share on the sector $s = 1, 2$. Consumer income I includes labor income wL (labor is the only factor of production), plus rebated tariff and tax revenue B, while free entry ensures that expected firm profits equal zero.

Domestic consumer demand for finished goods equals $\alpha_s(wL+B)$ in sector s. The finished good is also sold to firms in the same sector who are producing the differentiated intermediate inputs. To compute those sales, we start with the value of the finished good Y_s, which reflects the value of all intermediates – local and imported – that are bundled together. Let λ_{ds} denote the share of differentiated inputs that are purchased locally, where $\lambda_{d1} \leq 1$ in the traded sector 1 but $\lambda_{d2} = 1$ in the nontraded sector 2. Then $\lambda_{ds} Y_s$ is the value of locally-produced differentiated inputs. We also need to eliminate the markup on those inputs by dividing by $\sigma_s/(\sigma_s - 1)$ to obtain their costs of production, and then we take the share γ_s to obtain the value of finished goods that are sold as an input to firms. We denote the markup-adjusted cost share by

$$\tilde{\gamma}_s \equiv \gamma_s \rho_s \;\; \text{with} \;\; \rho_s \equiv \frac{(\sigma_s - 1)}{\sigma_s}. \tag{3}$$

The market clearing condition for the nontraded sector 2 is then $Y_2 = \alpha_2(wL + B) + \tilde{\gamma}_2 Y_2$ where $\lambda_{d2} \equiv 1$. In sector 1, the market clearing condition is more complex, with

$$Y_1 = \alpha_1(wL + B) + \tilde{\gamma}_1 \left(\lambda_{d1} Y_1 + \lambda_{x1} Y_1^*\right). \tag{4}$$

The term $\tilde{\gamma}_1 \lambda_{d1} Y_1$ on the right reflects the sale of the finished good to home firms. The next term, $\tilde{\gamma}_1 \lambda_{x1} Y_1^*$ reflects the home finished good *used in the production of the sector 1 differentiated inputs that are exported* (remember that the finished good is not directly exported). To obtain this term, we start with the *foreign* value of the finished good Y_1^*, and we define λ_{x1} as the home share of intermediate inputs that are bundled together to obtain Y_1^*. Then $\lambda_{x1} Y_1^*$ is the value of home exports of differentiated inputs, and once again we apply the parameter $\tilde{\gamma}_1$ to obtain the finished good that is sold to home firms to create those exports.

The expenditure shares are determined in equilibrium: in a heterogeneous firm model these depend on the optimal choice of cutoff productivities by firms, while in a homogeneous firm model the productivities are exogenously fixed at unity.[6] The cutoff productivities depend on the fixed costs of domestic production and exporting, and we assume that all fixed costs are paid in terms of labor in the source country, with the foreign wage chosen as the numeraire ($w^* \equiv 1$.)

To close the model, we need to use trade balance. As noted above, the term $\lambda_{x1}Y_1^*$ in (4) is the value of home exports of the differentiated inputs. Under balanced trade, this must equal the net-of-tariff value of imports. Letting t_1 denote one plus the *ad valorem* home import tariff in sector 1, the trade balance condition is then

$$\lambda_{x1}Y_1^* = \frac{\lambda_{m1}}{t_1}Y_1, \tag{5}$$

where λ_{m1} is the share of intermediate inputs used in sector 1 that are imported, with $\lambda_{d1}+\lambda_{m1} = 1$.[7]

As described by Demidova and Rodríguez-Clare (2013), the trade balance condition determines the wage w in the SOE, taking the foreign wage w^* as the numeraire. The equilibrium conditions of the SOE assume that changes in the tariff t_1 have a negligible impact on the foreign price index P_1^* and value of output Y_1^*. Fixing the values of P_1^* and Y_1^* means that the location of the foreign demand curve for a home exported variety is itself fixed, though that CES demand curve is not infinitely elastic as in a small-country *competitive* model. This means that trade policy has an impact on the small country's export price and therefore on its terms of trade.[8] We stress that the definition of a small open economy from Demidova and Rodríguez-Clare (2013) allows for a wide range of values for the home expenditure share, $0 < \lambda_{d1} < 1$, and likewise for its import share, $0 < \lambda_{m1} = 1 - \lambda_{d1} < 1$. A *special case* of the small open economy would be to consider $\lambda_{d1} \to 0$, so that the small country is importing nearly all of its intermediate inputs from abroad. We will not make use of this condition except as a limiting example after deriving our main results.

[6] The full equilibrium conditions are in Appendix A, Definition 1 for heterogeneous firms, and Appendix B, Definition 2 for homogeneous firms.

[7] Note that the import share is evaluated using the foreign export prices p_{x1}^* that are inclusive of the iceberg costs of trade, the markup, and the tariff t_1. In other words, we are assuming that the tariff is applied to the c.i.f. value of imports – including the markup. See further discussion in note 34. For simplicity, we assume no foreign tariff.

[8] To use the apt phrase of Bartelme, Costinot, Donaldson and Rodríguez-Clare (2019), a small country is "an economy that is large enough to affect the price of its own good relative to goods from other countries, but too small to affect relative prices in the rest of the world".

2.1 Response of Output and Entry to the Tariff

Before examining optimal policy, we describe the response of the finished good Y_1 and entry into each sector to the tariff. Using trade balance in (5), we can rewrite market clearing (4) as

$$Y_1 = \alpha_1(wL + B) + \tilde{\gamma}_1 \Lambda_1 Y_1, \text{ with } \Lambda_1 \equiv \left(\lambda_{d1} + \frac{\lambda_{m1}}{t_1}\right). \tag{6}$$

The first term on the right (6) is the demand for Y_1 as a final good, whereas the second term is the demand for Y_1 as an intermediate input, where Λ_1 equals the domestic share plus the *duty-free* import share. While this term is unity under either free trade ($t_1 = 1$) or autarky ($t_1 \to +\infty$ so $\lambda_{d1} = 1$ and $\lambda_{m1} = 0$), it has a lower value $\Lambda_1 < 1$ for all finite tariffs $t_1 > 1$.

We can simplify (6) by substituting for tariff revenue $B = \frac{t_1-1}{t_1}\lambda_{m1}Y_1$. Using $\lambda_{d1} + \lambda_{m1} = 1$, we can re-express tariff revenue as $B = (1 - \Lambda_1)\, Y_1$, and substituting above we obtain

$$Y_1 = \alpha_1[wL + (1 - \Lambda_1)Y_1] + \tilde{\gamma}_1 \Lambda_1 Y_1. \tag{7}$$

We see that starting at free trade, a tariff exerts two different forces on the value of the finished good, Y_1. On one hand, it raises tariff revenue $B = (1 - \Lambda_1)\, Y_1$ and increases consumer demand. On the other hand, it lowers duty-free imports and therefore lowers exports and Λ_1. Which of these forces dominates depends on the parameters α_1 and $\tilde{\gamma}_1$. We can readily solve for real output Y_1/w from (7) as

$$\frac{Y_1}{w} = \frac{\alpha_1 L}{\alpha_2 + (\alpha_1 - \tilde{\gamma}_1)\Lambda_1}. \tag{8}$$

We see that these two forces just offset each other when $\alpha_1 = \tilde{\gamma}_1$, in which case Y_1/w does not vary with the tariff. When $\alpha_1 > \tilde{\gamma}_1$, then consumer demand dominates and Y_1/w is a $\cap$-shaped function of the tariff (i.e., the inverse shape of Λ_1). In contrast, when $\alpha_1 < \tilde{\gamma}_1$ then exports dominate and Y_1/w is a $\cup$-shaped function of the tariff.

Substituting (8) back into $B = (1 - \Lambda_1)\, Y_1$, we obtain

$$\frac{B}{w} = \frac{\alpha_1 L\,(1 - \Lambda_1)}{1 - \tilde{\gamma}_1 - (\alpha_1 - \tilde{\gamma}_1)(1 - \Lambda_1)} = \frac{\alpha_1 L}{\frac{1-\tilde{\gamma}_1}{(1-\Lambda_1)} - (\alpha_1 - \tilde{\gamma}_1)}. \tag{9}$$

Because $\tilde{\gamma}_1 < 1$, from this final equation we see that B/w is monotonically decreasing in Λ_1, so their critical points are at the same tariff which we refer to as the *maximum (real) revenue* tariff. It follows from (8) that Y_1/w also has a critical point at that tariff.

The ambiguity in the shape of Y_1 does not extend to the entry of firms producing differentiated inputs in sector 1. Entry is proportional to the demand for those inputs for home sales, $\lambda_{d1}Y_1$, plus the demand for exports, $\lambda_{x1}Y_1^* = \frac{\lambda_{m1}}{t_1}Y_1$, which sum to $\Lambda_1 Y_1$. Using output in (8), entry is then

$$N_1^e = \frac{(\sigma_1 - 1)\Lambda_1 Y_1}{f_1^e \theta_1 \, \sigma_1} = \frac{\alpha_1(\sigma_1 - 1)}{f_1^e \theta_1 \, \sigma_1} \left[\frac{L}{\frac{\alpha_2}{\Lambda_1} + (\alpha_1 - \tilde{\gamma}_1)} \right]. \tag{10}$$

The ∪-shape for Λ_1 means that N_1^e is also ∪-shaped provided that $\alpha_2 > 0$: it falls as the tariff is increased from free trade, has a minimum at the maximum-revenue tariff, and then rises again to the same value in autarky and free trade. The intuition for this result is Lerner symmetry (Costinot and Werning, 2019): the import tariff acts like an export tax, and starting from free trade the tariff depresses entry into the traded sector and moves resources into the nontraded sector. In particular, entry into the nontraded sector is

$$N_2^e = \frac{\alpha_2(\sigma_2 - 1)}{f_2^e \theta_2 \, \sigma_2 (1 - \tilde{\gamma}_2)} \left(L + \frac{B}{w} \right),$$

which is a ∩-shaped function of the tariff because of tariff revenue. When $\alpha_1 = 1$ and there is no nontraded sector, however, then entry into the traded sector does not change with the tariff.

While the above equations have used the Pareto parameter θ_s from the heterogeneous firm model, the same conditions are obtained with homogeneous firms (see Appendix B) where we replace θ_s with:

$$\theta_s = \sigma_s^{hom} - 1. \tag{11}$$

Making this substitution in the equations above, the homogeneous firm model has the properties just discussed: in the presence of a nontraded sector, entry into the traded sector is *reduced* for an increase in the tariff starting from free trade, and then returns to its autarky value as the tariff becomes prohibitive. Condition (11) is familiar from Arkolakis, Costinot, and Rodríguez-Clare (2012), who demonstrate that the heterogeneous and homogeneous firm models are very similar in certain respects that include, as we have just argued, the impact of a tariff on entry.[9] We will see, however, that selection still plays a distinct role on the impact of a tariff with heterogeneous firms, especially in the presence of roundabout production.[10]

These results on entry contrast with the quite different results in the *firm-delocation* literature that combines a monopolistically competitive traded sector with a competitive *traded* outside good (see, e.g., Venables, 1987; Melitz and Ottaviano, 2008; Ossa, 2011), where a tariff *attracts* firms into the country applying it. In those models, the freely-traded outside good produced pins down the relative wage across countries, and a tariff on the monopolistically competitive imports *is not* the same as an export tax on those goods (since Lerner symmetry in this case implies that a uniform import tariff is equivalent to an export tax across *both* sectors). We return to this contrast in our concluding section.

3 Optimal Consumer and Producer Taxes in a Closed Economy

Before considering a tariff, we discuss the distortions arising in a closed economy from having monopolistic production of the differentiated inputs, where both sectors $s = 1, 2$ are nontraded. The markup on the differentiated inputs is fully passed-through to the price of the bundled, finished good. That distortion then operates on two margins: consumer purchases of finished goods; and firm purchases of finished goods as inputs, where the higher price on the finished good leads to inefficienctly low purchases of the finished good as compared to labor. Rather than correcting the monopoly distortion at its source (i.e., in the price of differentiated inputs), it will be instructive to explore how one could correct it by using taxes/subsidies on purchases of the finished goods on these two margins. So we consider both consumer and producer taxes/subsidies on purchases of the finished goods, where one plus the *ad valorem* rates are denoted by t_s^c and t_s^p, respectively.

With heterogeneous firms, the cutoff productivities chosen in each sector do not depend on these tax/subsidy instruments (see Appendix C). It follows that the optimal policy with homogeneous

[9] The full isomorphism between the models requires, however, that the fixed costs of exporting use resources of the destination country, as Arkolakis, Costinot, and Rodríguez-Clare (2012) explain. That is not our assumption, so there will be some differences between the models for this reason: see note 16.

[10] Note that Costinot and Rodríguez-Clare (2014) also find a distinct role for selection in the presence of multiple sectors and roundabout production, as we discuss in note 17.

firms or heterogeneous firms is identical. We consider two solutions to the closed-economy problem: first, choosing both the consumer and producer taxes/subsidies optimally; and second, using only the consumer tax/subsidy while setting $t_s^p \equiv 1$. When both instruments are used, we obtain the solution

$$t_s^p = \rho_s = \left(\frac{\sigma_s - 1}{\sigma_s}\right) < 1 \quad \text{and} \quad \frac{t_1^c}{t_2^c} = \frac{\rho_1}{\rho_2}. \tag{12}$$

The optimal producer subsidies $t_s^p < 1$ exactly counteract the markups on differentiated inputs which would otherwise be fully passed-through to finished goods prices.[11] With these subsidies, firms pay prices for finished goods that reflect their marginal costs. In addition, optimal consumption taxes/subsidies are needed so that, in relative terms, these prices offset the markups implicit in finished goods' prices faced by consumers.

In contrast to this first-best case, consider the second-best policy that involves consumption taxes/subsidies only. In that case, the distortion that arises from having a high price of the finished good as an input (due to the markup on differentiated inputs that is fully passed-through to the finished good price) is not corrected. It is instructive in this case to solve for the price of the finished good. That price index, in the absence of imports, is

$$P_s = \left(N_s^e \int_{\varphi_{ds}}^{\infty} p_{ds}\left(\varphi\right)^{1-\sigma_s} g_s\left(\varphi\right) d\varphi \right)^{\frac{1}{1-\sigma_s}} \; = \; \left(N_{ds}\right)^{\frac{-1}{(\sigma_s - 1)}} \left(\frac{\sigma_s}{\sigma_s - 1}\right) \frac{c_s}{\bar{\varphi}_s}, \tag{13}$$

where N_s^e is the mass of entering firms and φ_{ds} is the cutoff productivity to remain in the market, while $N_{ds} = N_s^e[1 - G_s(\varphi_{ds})]$ is the mass of surviving firms (equal to domestic product variety) and $\bar{\varphi}_{ds}$ is their average productivity,[12] so that $\left(\frac{\sigma_s}{\sigma_s - 1}\right) \frac{c_s}{\bar{\varphi}_s}$ is their average price. We now substitute from the input cost index in (1) to solve for the price index,

$$P_s = \left(N_{ds}\right)^{\frac{-1}{(1-\gamma_s)(\sigma_s - 1)}} w \left(\frac{\sigma_s}{\sigma_s - 1} \frac{1}{\bar{\varphi}_s}\right)^{\frac{1}{(1-\gamma_s)}}. \tag{14}$$

Notice that the impact of product variety on reducing the price index has increased from $1/(\sigma_s - 1)$ in (13) to $1/(1 - \gamma_s)(\sigma_s - 1)$ in (14). That change carries through to other equations for

[11]The need for such subsidies in a dynamic monopolistic competition model was noted by Judd (1997, 2002).

[12]This average productivity is defined as in Melitz (2003) and equals $\bar{\varphi}_{ds} = \varphi_{ds} \left(\frac{\theta_s}{\theta_s - \sigma_s + 1}\right)^{\frac{1}{\sigma_s - 1}}$.

the closed economy equilibrium, so the economy with roundabout production is effectively acting like a closed economy *without* roundabout but with a lower elasticity of substitution, defined by

$$\tilde{\sigma}_s \equiv 1 + (1 - \gamma_s)(\sigma_s - 1) < \sigma_s. \tag{15}$$

A change in entry in (14) changes the price index with the exponent $1/(\tilde{\sigma}_s - 1)$, so that $\tilde{\sigma}_s$ is the *effective* elasticity of substitution. We find the second-best optimal consumption taxes/subsidies (see Appendix C) are given by

$$\frac{t_1^c}{t_2^c} = \left(\frac{\tilde{\sigma}_1 - 1}{\tilde{\sigma}_1}\right) \Big/ \left(\frac{\tilde{\sigma}_2 - 1}{\tilde{\sigma}_2}\right). \tag{16}$$

Notice that the consumption taxes/subsidies in (16) are similar to those in (12), but are now evaluated using the effective elasticities of substitution: the sector with the lowest effective elasticity must have the lowest tax (i.e., greatest subsidy) to offset the effective monopoly distortion. Even if the elasticities $\sigma_s \equiv \sigma > 1$ are identical then the sector with the stronger roundabout production (higher γ_s) will have the lower effective elasticity in (15) and should be subsidized. The role of the effective elasticities in this second-best case for a closed economy will be useful as we examine tariffs on trade, to which we turn next.

4 First-best Uniform Tariff in a Small Open Economy

Demidova and Rodríguez-Clare (2009) analyze a SOE with one sector and no roundabout production. They identify two distortions arising from monopolistic competition. The first is the markup charged on the domestic differentiated varieties which can be corrected by subsidizing domestic buyers of those inputs, where one minus the *ad valorem* subsidy is set equal to the inverse of the markup,

$$t_1^d = \rho_1 = \frac{\sigma_1 - 1}{\sigma_1}. \tag{17}$$

Alternatively, the markup on domestic varieties can be offset by using a tariff on imported varieties equal to the markup, $t^{hom} = \frac{1}{\rho_1} = \frac{\sigma_1}{\sigma_1 - 1}$, which offsets the domestic markup in relative terms by introducing the same distortion on import prices. This is the optimal tariff in a one-sector SOE with monopolistic competition and homogeneous firms (Gros, 1987).

With heterogeneous firms, however, Demidova and Rodríguez-Clare (2009) find that there is a second distortion: each new foreign variety brings surplus, which domestic buyers do not take account of in their spending. One way to correct this externality is to use an import subsidy, and they find that one minus the optimal *ad valorem* subsidy is

$$t_1^m = \frac{\theta_1 \rho_1}{(\theta_1 - \rho_1)} < 1, \tag{18}$$

where the inequality follows from $\theta_1 > \sigma_1 - 1$. So the first-best is achieved by using the two subsidies t_1^d, $t_1^m < 1$. Furthermore, they argue that that an *equivalent* policy is to multiply the tariff $t^{hom} = \frac{1}{\rho_1}$ by the import subsidy in (18), and then both distortions are corrected by a single instrument, which is the optimal tariff in a one-sector model with heterogeneous firms,

$$t^{het} \equiv t^{hom} \times t_1^m = \frac{\theta_1}{(\theta_1 - \rho_1)} > 1. \tag{19}$$

It is immediate that $t^{het} < t^{hom}$ when evaluated with the same parameter σ_1, since $t_1^m < 1$.[13] But even when we do the more exact comparison across models, then we still find that $t^{het} < t^{hom}$ because $\theta_1 > \sigma_1^{het} - 1$ and so using (11), $\sigma_1^{hom} > \sigma_1^{het}$ and $\rho_1^{hom} > \rho_1^{het}$.[14]

If we add a second sector or roundabout production, then the equivalence of using the policy $t_1^d, t_1^m < 1$ and the optimal tariff $t^{het} > 1$ no longer holds, however. To see this, suppose that we "scaleup" t_1^d, t_1^m by dividing by ρ_1 (i.e., multiplying by $\frac{\sigma_1}{\sigma_1 - 1}$), thereby obtaining $t_1^d = 1$ and the import tariff of t^{het}, and then use a subsidy of ρ_1 on the finished good to *offset* this scaling-up. With a single sector and no roundabout production, this subsidy does not make any difference because consumers cannot substitute away from the finished good and firms do not purchase it. But once we add multiple sectors and/or roundabout production, then substitution by consumers and firms means that the subsidy of ρ_1 is needed to avoid the downstream impact of the markup $\frac{\sigma_1}{\sigma_1 - 1}$, as we found in the closed economy. In general, for an open economy with multiple sectors and input-output linkages, Lashkaripour and Lugovsky (2020) argue that such subsidies must be

[13] The same small-country formula for the optimal tariff as (19) is obtained by Felbermayr, Jung and Larch (2013), who show that the optimal tariff in a large country is higher.

[14] From (11) and (19) we then have $t^{het} = 1/[1 - (\rho_1^{het}/\theta_1)] < 1/[1 - (\rho_1^{hom}/\theta_1)] = 1/[1 - (1/\sigma_1^{hom})] = t^{hom}$.

applied in the first-best: in that case, the first-best tariffs for a *small* country are the same with and without input-output linkages.[15] Our interest is in the second-best tariff obtained in the absence of such subsidies, as we turn to next.

5 Second-Best Uniform Tariff in a Small Open Economy

We now add the nontraded sector 2, which can also have roundabout production, and we suppose that the *only* policy instrument available is a uniform import tariff (or subsidy) t_1 with an optimal second-best value t_1^*. The fact that a subsidy on the finished good is not used creates a robust reason for lowering the optimal tariff below t^{het}. A slight reduction of the tariff below its first-best value ordinarily causes only a second-order loss in welfare, but it now brings a first-order gain in welfare because it lowers the price of the finished good purchased by firms. There are two other reasons to have $t_1^* < t^{het}$, which arise from the response of wages and the response of entry to changes in the tariff. We consider each of these in the following sections.

5.1 Response of the Home Wage to the Tariff

A key insight of Demidova and Rodríguez-Clare (2013) is that in the monopolistic competition model, even a SOE experiences an increase in its wage from applying a tariff. That wage increase results in a rise in its export prices, which is analogous to the terms of trade effect of a tariff that occurs in competitive models. Lashkaripour (2021) stresses the importance of this wage elasticity in determining the welfare impact of tariffs changes, and therefore the Nash-equilibrium tariffs in his model.

When solving for the impact of the tariff on wages, we would like to compare the solutions with homogeneous firms and heterogeneous firms, and also understand the impact of roundabout production in either case. We begin by examining the trade balance condition (5) in the homogeneous firm model, where the export share λ_{x1} equals

$$\lambda_{x1} = N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c_1 \, \tau_{x1}}{P_1^*} \right)^{1-\sigma_1}, \tag{20}$$

[15] See their section 4(ii) and especially footnote 23, which explains that for a small open economy the equations for the first-best taxes and tariffs are identical with and without input-output linkages.

where N_1^e is the endogenous entry of firms into sector 1, τ_{x1} are iceberg trade costs, and P_1^* is the foreign price index in sector 1 which is exogenous for the SOE. The import share λ_{m1} equals

$$\lambda_{m1} = N_1^{e*} \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c_1^* \, \tau_{x1}^* \, t_1}{P_1} \right)^{1-\sigma_1}, \tag{21}$$

where N_1^{e*} is the entry of foreign firms into sector 1 and c_1^* are their input costs, both of which are exogenous. The home price index P_1 is endogenous, but given its value then an increase in the tariff t_1 reduces the import share, and reduces the tariff-free import share λ_{m1}/t_1 even more. Given Y_1, then to satisfy trade balance this reduction in duty-free imports $\lambda_{m1}Y_1/t_1$ would need to be matched by a reduction in exports $\lambda_{x1}Y_1^*$. That can be achieved by an increase in home wages, which raise the input costs c_1 in (20). This reasoning illustrates the positive terms of trade impact of a tariff in the SOE, but it needs to be sharpened to take into account the endogenous price index P_1 and also roundabout production.

To solve for P_1, we proceed indirectly by focusing on the *domestic* share, which equals

$$\lambda_{d1} = N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c_1}{P_1} \right)^{1-\sigma_1}. \tag{22}$$

Inverting this equation we obtain an expression for the sector 1 price index,

$$P_1 = \left(\frac{\lambda_{d1}}{N_1^e} \right)^{\frac{1}{(\sigma_1 - 1)}} \frac{\sigma_1 c_1}{(\sigma_1 - 1)}. \tag{23}$$

Replacing the domestic share λ_{d1} by $1 - \lambda_{m1}$ in this expression, and also substituting from (1), we solve for the price index as

$$P_1 = \left(\frac{1 - \lambda_{m1}}{N_1^e} \right)^{\frac{1}{(1-\gamma_1)(\sigma_1 - 1)}} w \left(\frac{\sigma_1}{\sigma_1 - 1} \right)^{\frac{1}{(1-\gamma_1)}}. \tag{24}$$

Notice that the impact effect of the tariff on reducing the import share λ_{m1} will increase the price index P_1, and this index is increasingly sensitive to the import share as the extent of roundabout production grows, so that γ_1 rises.

Substituting P_1 back in the input cost index in (1), and totally differentiating, we obtain

$$\hat{c}_1 = \hat{w} - \frac{1}{(\sigma_1 - 1)} \left(\eta_{m1} \hat{\lambda}_{m1} + \frac{\gamma_1 \hat{N}_1^e}{(1-\gamma_1)} \right) \text{ where } \eta_{m1} \equiv \frac{\gamma_1 \lambda_{m1}}{(1-\gamma_1)(1-\lambda_{m1})}. \tag{25}$$

Intuitively, after the impact effect of the tariff on reducing duty-free imports, think of the equilibrium being restored by a rise in the input costs c_1, which reduces exports. In the absence of roundabout production, the rise in c_1 is achieved by an increase in the wage. With roundabout, however, we see from (25) that the fall in the import share itself – by raising the price index in (24) – contributes to restoring equilibrium, so that a *smaller* increase in the wage is needed. The coefficient η_{m1} on $\hat{\lambda}_{m1}$ in (25) is an endogenous variable that depends on the import share, and it is increasing in the amount of roundabout production γ_1. By this argument, the wage impact of the tariff is moderated by the extent of roundabout production, as will be confirmed below. In addition, notice that the induced exit from sector 1 – as we discussed in section 2.1 – also moderates the increase in the wage needed to obtain a given rise in c_1.

The argument we have just made on how roundabout production reduces the terms of trade impact of the tariff applies with heterogeneous firms, too, in which case selection effects come into play. The above equation for the change in marginal costs (see Appendix A.6) then becomes

$$\hat{c}_1 = \hat{w} - \frac{1}{\theta_1} \left(\eta_{m1} \hat{\lambda}_{m1} + \frac{\gamma_1 \hat{N}_1^e}{(1-\gamma_1)} + \frac{\gamma_1(\theta_1 - \sigma_1 + 1)}{(1-\gamma_1)(\sigma_1 - 1)} (\hat{Y}_1 - \hat{w}) \right). \tag{26}$$

Using the parameter restriction (11), we see that the the fall in the import share has the same impact in (25) and (26), and reduces the increase in the wage needed to restore equilibrium. Induced exit from sector 1 also moderates the increase in the wage. In addition, a third term appears on the right of (26), and that is the change in real output Y_1/w. Recall from our discussion in section 2.1 that an increase in the tariff from free trade increases (decreases) the real value of output Y_1/w when $\alpha_1 > (<)\ \tilde{\gamma}_1$. The presence of this term in (26) reflects the selection effect of real output on the cutoff productivity for domestic firms. In particular, when roundabout production is strong enough so that $\tilde{\gamma}_1 > \alpha_1$ and $\hat{Y}_1 - \hat{w} < 0$, then this selection effect in the domestic market increases the cutoff, reduces product variety and increases the price index, further moderating the increase in the wage needed to restore equilibrium to the trade balance. This result is our first illustration of how selection due to heterogeneous firms – in conjunction with roundabout production – influences the impact of a tariff.

There is another selection effect that also reduces the terms of trade impact of the tariff with heterogeneous firms, even in the absence of roundabout production. Consider the share of home exporters in the foreign market, which is

$$\lambda_{x1} = \varphi_{x1}^{-\theta_1} N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c_1 \, \tau_{x1}}{\bar{\varphi}_{x1} \, P_1^*} \right)^{1-\sigma_1}, \tag{27}$$

where $\varphi_{x1} > 1$ is the cutoff productivity for home exporters with average productivity $\bar{\varphi}_{x1}$. The first terms on the right, $\varphi_{x1}^{-\theta_1} N_1^e$, equals the mass of exported varieties and is influenced by the selection of exporters. By solving for the cutoff productivity (see Appendix A.6), we obtain an alternative expression for the export share,

$$\lambda_{x1} = N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c_1 \, \tau_{x1}}{P_1^*} \right)^{-\theta_1} \left(\frac{\sigma_1 w f_{x1}}{Y_1^*} \right)^{1 - \frac{\theta_1}{(\sigma_1 - 1)}} \left(\frac{\theta_1}{\theta_1 - \sigma_1 + 1} \right). \tag{28}$$

This expression is very similar to the export share in the homogeneous firm model in (20), except for the middle term on the right of (28), involving w/Y_1^*, that reflects the selection of home exporters. The rise in wages from a tariff increases this middle term, which raises the cutoff productivity for home exporters and reduces their export share. This selection effect works in the direction of restoring equilibrium in the trade balance, and therefore reduces the increase in wages needed for equilibrium.[16] This is our second illustration of how selection influences the impact of a tariff.

To summarize, we have argued the impact of the tariff on home wages is reduced when there is roundabout production, and reduced when firms are heterogeneous. To confirm these results, we solve for the marginal impact of the tariff and sector 1 entry on the wage (see Appendix D.1) for either homogeneous or heterogeneous firms as denoted by the superscript z, writing this as

$$\hat{w} = \mathcal{E}_1^z(\gamma_1)\hat{t}_1 + \mathcal{E}_2^z(\gamma_1)\hat{N}_1^e, \quad \text{for } z = hom, het, \tag{29}$$

[16]This extra impact of a tariff due to selection arises from our assumption that the fixed costs of exporting use domestic labor rather than using foreign labor (whose wage is fixed as the numeraire). Likewise, when foreign firms pay their fixed costs of exporting using their own labor, then there is an extra impact of selection on the import share at home, as discussed in Appendix A.6. When we make the alternative assumption that the fixed costs of exporting use labor in the destination country, then these two extra impacts disappear.

where $\hat{N}_1^e$ denotes the change in entry into sector 1 and the elasticities $\mathcal{E}_n^z(\gamma_1)$, $n = 1, 2$ are the marginal impact of the tariff and entry on the wage that depend on $\gamma_1 \in [0, 1)$ and the market structure $z = hom, het$ (as well as on other parameters and the endogenous import share). With only a single sector, $\alpha_1 = 1$, the tariff has no impact on entry in sector 1 so that $\hat{N}_1^e = 0$. When evaluating at free trade for simplicity, so that $t_1 = 1$, then we can compare the marginal impact of the tariff on wages depending on the amount of roundabout production.

We confirm (see Appendix D.1) that with either homogeneous or heterogeneous firms, an increase in the extent of roundabout production moderates the wage impact of the tariff, a result we state as:

$$\begin{aligned}\text{For } \alpha_1 = t_1 = 1 \text{ and } \sigma_1 > 2: \quad & \mathcal{E}_1^z(0) > 0 \text{ and } \mathcal{E}_1^z(\gamma_1) \text{ is declining in } \gamma_1, \text{ with } \mathcal{E}_1^{hom}(\gamma_1) < 0 \\ & \text{for } \eta_{m1} > \tfrac{\sigma_1}{\sigma_1 - 2} \text{ and } \mathcal{E}_1^{het}(\gamma_1) < 0 \text{ for } \eta_{m1} > \tfrac{\sigma_1}{\sigma_1 - 2 + \lambda_{1m}}. \end{aligned} \tag{30}$$

As expected from our arguments above, the marginal impact of the tariff on the wage is *reduced* by the extent of roundabout production (which in this statement is a parametric increase in γ_1 while holding the import share constant). Surprisingly, we find that $\mathcal{E}_1^z(\gamma_1) < 0$ so the wage *falls* rather than rises with the tariff when the extent of roundabout and the import share – as reflected by η_{m1} – are sufficiently large. This occurs because of the large impact of the reduced import share on the price index P_1 and therefore the input costs in (25) and (26), so that a *fall* in the wage is needed to restore equilibrium. In that case, an import *subsidy* rather than a tariff would be needed to raise the home wage. We will explore in later results whether an import subsidy can be the optimal second-best policy.

We compare across the two market structures using the parameter restriction in (11) (and assuming the same import share under free trade), with no roundabout production for simplicity, to obtain:

$$\text{For } \alpha_1 = t_1 = 1, \sigma_1 > 2 \text{ and using (11):} \quad \mathcal{E}_1^{het}(0) < \mathcal{E}_1^{hom}(0). \tag{31}$$

This result shows the impact of selection in reducing the terms of trade effect in the heterogeneous firm model, and by continuity it continues to hold for a range of positive values for γ_1.

5.2 Entry and Welfare

Aside from its reduced impact on the wage, another reason for the tariff to be lower in a second-best setting is through changing the entry of firms. Starting from free trade we found in section 2.1 that an increase in t_1 from free trade leads to the *exit* of firms from the traded sector 1 and entry into the nontraded sector 2. To solve for the impact of that exit and entry on welfare, we start with indirect utility corresponding to (2), which is (up to a constant): $U = (wL + B)/(P_1^{\alpha_1} P_2^{\alpha_2})$. We totally differentiate utility for a change in the tariff, using the expressions for the price indexes (see Appendix D.2), to obtain

$$\begin{aligned}\hat{U} \;=\; & -\frac{\alpha_1}{\theta_1(1-\gamma_1)}\hat{\lambda}_{d1} + \sum_{s=1,2}\alpha_s\left[1+\frac{(1-\Gamma_s)}{\theta_s(1-\gamma_s)}\left(\frac{\theta_s}{\sigma_s-1}-1\right)\right]\frac{B}{wL+B}(\hat{B}-\hat{w}) \\ & + \sum_{s=1,2}\alpha_s\left[\frac{(1-\Gamma_s)}{\theta_s(1-\gamma_s)}+\frac{\Gamma_s}{(\sigma_s-1)(1-\gamma_s)}\right]\hat{N}_s^e, \end{aligned} \tag{32}$$

where $\Gamma_1 \equiv \tilde{\gamma}_1 \Lambda_1$ denotes the fraction of the sector 1 finished good used as an input in (8), with $\Gamma_2 \equiv \tilde{\gamma}_2$, and $1 - \Gamma_s = \alpha_s (wL + B)/Y_s$ is the fraction used as a final good in each sector. Note that Γ_s is another way to measure the extent of roundabout production in a sector.

The first term in (32) is the change in the domestic share in sector 1 and is familiar from Arkolakis, Costinot, and Rodríguez-Clare (2012), where it is a sufficient statistic for the welfare change due to a change in iceberg trade costs in a one-sector model with no roundabout. Using a tariff introduces the second term in (32), reflecting the change in real tariff revenue B/w. Most important for our purposes is the third term in (32), which is related to entry. If there is no roundabout production so $\gamma_s = \Gamma_s = 0$, then the third term is simply the weighted sum of $\hat{N}_s^e/\theta_s$ across sectors using the weights $1/\theta_s(1-\gamma_s)$ that appear in the first term. When there is roundabout production, however, then a new mechanism comes into play. The effect of entry in the final term of (32) now depends on Γ_s, the fraction of finished output used as an intermediate input. The coefficient of that term is $1/(\sigma_1 - 1)(1 - \gamma_s)$, which *exceeds* $1/\theta_s(1 - \gamma_s)$ because $\theta_s > \sigma_1 - 1$. It follows that when the finished output arising from new entry is used more heavily downstream as an *intermediate input to other firms*, rather than just sold to consumers (in which case $\Gamma_s = 0$), then these forward linkages create a magnified effect of entry on welfare.[17]

These results can be contrasted to the case with homogeneous firms. Then using the parameter restriction (11), the weights appearing in the final bracketed term in (32) are both replaced by $1/(\sigma_s^{hom}-1)(1-\gamma_s)$, so this final term would appear as

$$\sum_{s=1,2}\alpha_s\left[\frac{(1-\Gamma_s)}{(\sigma_s^{hom}-1)(1-\gamma_s)}+\frac{\Gamma_s}{(\sigma_s^{hom}-1)(1-\gamma_s)}\right]\hat{N}_s^e \;=\; \sum_{s=1,2}\frac{\alpha_s\hat{N}_s^e}{(\sigma_s^{hom}-1)(1-\gamma_s)}. \tag{33}$$

We see that entry under homogeneous firms has the same impact whether the finished good is used as an intermediate input or a final good, so the share Γ_s no longer appears. Comparing (33) with the final bracketed term of (32), we also see that in both cases the welfare impact of entry depends on $1/(1-\gamma_s)$, so that entry into sectors with more roundabout production (higher γ_s) will have a greater welfare benefit – holding constant the other parameters. But with heterogeneous firms, the finished output arising from new entrants that is used as an *intermediate input* has a *magnified* impact in (32) when $\Gamma_s > 0$, because $\theta_s > \sigma_s - 1$. These results are a third and final illustration of how selection with heterogeneous firms influences the welfare impact of a tariff.[18]

To fully solve for the impact of entry and the tariff on welfare, we focus the remainder of the paper on the heterogeneous firm model: from our arguments above, we are therefore focusing on the case with the greatest potential to lower the second-best tariff. Using the change in the tariff and in wages to compute the change in the expenditure share $\hat{\lambda}_{d1}$ in (32), and also solving for the change in tariff revenue, we obtain the following reduced-form expression for the change in welfare due to selection and entry:[19]

$$\hat{U} = \alpha_1\left[\mathcal{E}_\varphi\;\hat{\varphi}_{x1} + D(t_1)\hat{N}_1^e\right], \tag{34}$$

[17]We stress that a weighted sum of the log changes in entry across sectors (using their labor shares as weights) equals zero, as we show in Appendix D.2. So utility can rise only if the beneficial impact of entry in one sector exceeds the cost from reduced entry in the other.

[18]Because this difference between the results with homogeneous and heterogeneous firms arises even when we impose the parameter restriction (11), it shows that the two models are not isomorphic in the presence of roundabout production when entry is changing across sectors, as also found by Costinot and Rodríguez-Clare (2014): compare columns 5 and 6 of their Table 4.3 (p. 232).

[19]Note that the elasticity $\mathcal{E}_\varphi$ incorporates changes in φ_{x1} and all other cutoffs, while $D(t_1)$ incorporates the change in both N_1^e and N_2^e. In addition, (34) incorporates the change in the wage and in tariff itself, which is inverted so that it is a function of $\hat{\varphi}_{x1}$ and $\hat{N}_1^e$: see Appendix D.4.

where

$$D(t_1) \equiv \left[\frac{\tilde{\sigma}_1}{(\tilde{\sigma}_1 - 1)} - \frac{\tilde{\sigma}_2}{(\tilde{\sigma}_2 - 1)} \frac{\Lambda_1(1-\tilde{\gamma}_1)}{1-\tilde{\gamma}_1\Lambda_1} - \mathcal{E}_d \right]. \tag{35}$$

To interpret (34), the first term appearing on the right in brackets summarizes all the selection effects from the change in the tariff. The second term is the change in sector 1 entry $\hat{N}_1^e$ times $D(t_1)$, which denotes the marginal welfare impact of entry into the traded sector – holding the cutoff productivities constant – relative to the size of that sector (α_1). From (35), the marginal impact of entry equals the terms: $\frac{\tilde{\sigma}_1}{(\tilde{\sigma}_1-1)}$, which is the effective distortion in sector 1; minus the effective distortion in sector 2 multiplied by $\frac{\Lambda_1(1-\tilde{\gamma}_1)}{1-\tilde{\gamma}_1\Lambda_1}$ (which is ≤ 1 for $t_1 \geq 1$) that reflects tariff revenue; minus the term $\mathcal{E}_d > 0$ that we interpret as the deadweight loss of the tariff, which is an inefficient instrument to influence entry.[20]

We see from (35) that $D(t_1) > 0$ so that entry into the traded sector leads to a welfare gain – and exit leads to a welfare loss – when that effective distortion there is *sufficiently above* the effective markup in the nontraded sector. We want to allow the effective distortion in the traded sector to be greater or less than that in the nontraded sector, but we do not want the latter distortion to be too high. Accordingly, we will impose an upper-bound on the distortion in the nontraded sector,

$$\frac{\tilde{\sigma}_2}{(\tilde{\sigma}_2 - 1)} < \kappa_0 + \kappa_1 \frac{\tilde{\sigma}_1}{(\tilde{\sigma}_1 - 1)}, \tag{36}$$

where the parameters κ_0, κ_1 will be specified in Theorem 1 below. Our aim is to allow for a wide range of effective distortions in (36).

5.3 Optimal Second-Best Tariff

We can now state a general formula for the optimal second-best tariff t_1^*, as compared to t^{het} (see Appendix E). Specifically, t_1^* is obtained as a fixed point of the equation

$$t_1^* = t^{het}\, F(t_1^*), \text{ with } F(t_1) \equiv \left[\frac{1-\gamma_1 R(t_1)}{1+\alpha_2 M(t_1)} \right], \tag{37}$$

[20] All script variables $\mathcal{E}_n$, $n = \varphi, d, a, m$ depend on sector 1 parameters including γ_1 and λ_{d1} and therefore depend on the tariff. They are defined in Appendixes D.4 and D.5.

where $M(t_1)$ captures the impact of the higher monopoly distortion in the traded versus the nontraded sectors, and is defined by

$$M(t_1) \equiv \mathcal{M} \times \left(\mathcal{E}_m - \frac{(t_1 - 1)}{t_1} \theta_1 \right) \frac{D(t_1)}{A(t_1)} \quad \text{with} \quad \mathcal{M} > 0,\ \mathcal{E}_m > 0, \tag{38}$$

and $A(t_1)$ is defined by

$$A(t_1) \equiv \alpha_1 - \tilde{\gamma}_1 + \alpha_2 \mathcal{E}_a \quad \text{with} \quad \mathcal{E}_a > 0, \tag{39}$$

while $R(t_1)$ reflects the impact of roundabout production and is defined by

$$R(t_1) = \mathcal{R} \times \left[\frac{\theta_1 - \rho_1 (1 - \lambda_{d1})}{\Lambda_1} - \theta_1 \rho_1 \right], \tag{40}$$

with

$$\mathcal{R} = \left\{ \lambda_{d1} \tfrac{\theta_1}{(\sigma_1 - 1)} \left(\tfrac{\theta_1}{\sigma_1 - 1} - \tfrac{1}{\sigma_1} \right) \left[(\tilde{\sigma}_1 - 1) \left(1 + \tfrac{\sigma_1}{\Lambda_1} \right) + 1 \right] \right\}^{-1} > 0. \tag{41}$$

To explain these terms, recall that the distortion term $D(t_1)$ measures the marginal welfare impact of firms moving from the nontraded to the traded sector, and notice that it enters $\alpha_2 M(t_1)$, which appears in the denominator of (37), reflecting the impact of the *relative* monopoly distortion on the optimal tariff. When $\alpha_1 = 1$ so there is only the traded sector, then this term vanishes because there is no impact of the *relative* distortion between traded and nontraded goods. But there is still a monopoly distortion in traded goods alone, where the markup distorts the use of the finished good as an input relative to labor. Notice that $\mathcal{R} > 0$ in (41) is declining in the effective elasticity $(\tilde{\sigma}_1 - 1) = (\sigma_1 - 1)(1 - \gamma_1)$, so as that elasticity falls then the term $R(t_1)$ in the numerator of (37) rises, which tends to pull down the optimal tariff. This illustrates a complementary relationship between roundabout production and the monopoly distortion in the traded sector in reducing the optimal tariff. If there was not monopoly distortion, then we would have $\mathcal{R} = 0$ and the presence of roundabout production would not matter for the optimal tariff.[21]

When $\alpha_1 = 1$ and $\gamma_1 = 0$ in (37), then we are back in the one-sector, no-roundabout model and that formula immediately gives $t_1^* = t^{het}$. Outside of that special case, there will be a lower optimal

[21] Holding fixed the ratios $\theta_1/(\sigma_1 - 1)$ in (41), we see that as $\sigma_1 \to +\infty$ then $\mathcal{R} \to 0$, so that roundabout production does not have any impact on the optimal tariff when the differentiated inputs become very strong substitutes and the monopoly distortion in the traded sector vanishes.

tariff, $t_1^* < t^{het}$, whenever $\alpha_2 M(t_1^*) \geq 0$ and $\gamma_1 R(t_1^*) \geq 0$ with one of these inequalities holding strictly. For example, suppose that $\alpha_1 = 1$ so there is only a traded sector, but $\gamma_1 > 0$ so there is some roundabout production. Then we prove below that $R(t_1^*) > 0$ at the fixed point of (37), so that roundabout production lowers the optimal tariff. Thus, we will find that the optimal tariff is lowered by the monopoly distortion in sector 1, even in the absence of the nontraded sector.

Next, suppose we add the nontraded sector so that $\alpha_2 > 0$, in which case the denominator of $F(t_1^*)$ which is $[1 + \alpha_2 M(t_1^*)]$ comes into play. If the relative distortion in the traded sector is positive, $D(t_1^*) > 0$, then provided that the other terms in (38) are positive we will have $M(t_1^*) > 0$, so the denominator *further reduces* the optimal tariff. One of those other terms is $A(t_1)$. Recall that we defined $D(t_1)$ in (35) as the marginal impact of entry into sector 1 relative to the size of that sector (α_1), and we loosely interpret $A(t_1)$ as the effective size of sector 1. As a regularity condition we need to impose $A(t_1) > 0$, which is guaranteed by the sufficient conditions specified in the following main theorem (proved in Appendix E).

Theorem 1.

(a) ***Pure roundabout:*** *If $\alpha_1 = 1$ and $\gamma_1 > 0$, then $R(t_1^*) > 0$ and the optimal tariff is $t_1^* < t^{het}$.*

(b) ***No roundabout:*** *If $\gamma_1 = \gamma_2 = 0$ then (i) $D(t_1^*) > 0$ and the optimal tariff is $t_1^* < t^{het}$ when*

$$\frac{\sigma_2}{(\sigma_2 - 1)} < \frac{\sigma_1}{(\sigma_1 - 1)} - \frac{1}{\theta_1}. \tag{42}$$

(ii) If $\frac{\sigma_2}{(\sigma_2-1)} \geq t^{het} \frac{\sigma_1}{(\sigma_1-1)}$, then $D(t_1^*) < 0$ and the optimal tariff is $t_1^* > t^{het}$.

(c) ***Two sectors with roundabout:*** *Assume that $\alpha_2 > 0$ and the following two conditions hold:*

$$\gamma_1 \leq \frac{\frac{\sigma_1}{\rho_1}(\theta_1 - \rho_1)(1 - \rho_1)}{1 + \frac{\sigma_1}{\rho_1}(\theta_1 - \rho_1)(1 - \rho_1)}, \tag{43}$$

$$\alpha_2 \leq \max\left\{1 - \tilde{\gamma}_1, \frac{\frac{\theta_1(1-\rho_1)}{\rho_1} + (1-\gamma_1)\theta_1}{\frac{\theta_1(1-\rho_1)}{\rho_1} + \rho_1\left(1 + \frac{\gamma_1}{\sigma_1(1-\gamma_1)}\right)}\right\}. \tag{44}$$

Then $A(t_1) > 0$ for $t_1 > t_1'$, where $t_1' < 1$ is an import subsidy. Furthermore, if there is enough *roundabout production so that*

$$\gamma_1 \geq \frac{\rho_1}{\left[\theta_1(1-\rho_1)+\rho_1^2\right](\theta_1-\rho_1)}, \tag{45}$$

and the upper bound in (36) holds as

$$\frac{\tilde{\sigma}_2}{(\tilde{\sigma}_2-1)} < \frac{(t^{het}-\tilde{\gamma}_1)}{(1-\tilde{\gamma}_1)}\frac{\tilde{\sigma}_1}{(\tilde{\sigma}_1-1)} + \kappa_0, \tag{46}$$

where κ_0 *is independent of the share* λ_{d1}*,*[22] *then the optimal tariff is* $t_1^* < t^{het}$ *with* $R(t_1^*) > 0$*.*

The proof of Theorem 1 does not use the fixed-point formula (37) directly, but rather, uses a slight transformation of it. Taking the difference between the numerator of $F(t_1)$ times t^{het} and the denominator times t_1, we obtain

$$H(t_1) \equiv t^{het}\left[1-\gamma_1 R(t_1)\right] - t_1\left[1+\alpha_2 M(t_1)\right]. \tag{47}$$

The function $H(t_1)$ is a continuous function of the tariff provided that $A(t_1) > 0$ in the interval of tariffs we are interested in, in which case $M(t_1)$ will not have any discontinuities. Our approach for each part of Theorem 1 is to find high and low tariffs at which the sign of $H(t_1)$ switches, and then we apply the intermediate value theorem to obtain a point where $H(t_1^*) = 0$, which by construction is a fixed-point of (37) so that t_1^* is the optimal tariff.

Part (a) of Theorem 1 shows that roundabout production in a one-sector model always lowers the optimal tariff. This result is the simplest demonstration that the tariff t_1^* on intermediate inputs is less than the tariff t^{het} that applies in a model with differentiated final goods. To prove this result, we note that with $\alpha_1 = 1$ then $M(t_1)$ disappears from (47) because there is no monopoly distortion *between* sectors, and we only need to work with the term $R(t_1)$ that incorporates roundabout production and the monopoly distortion *within* sector 1. We first establish (see Appendix E.1) that at $t^{het} > 1$ then $R(t^{het}) > 0$, so that we obtain $H(t^{het}) = -t^{het}\gamma_1 R(t^{het}) < 0$ for $\gamma_1 > 0$. Next, we establish that there is a low enough tariff $t^{R0} < 1$ at which $R(t^{R0}) = 0$, which means that the

[22]The formula for κ_0 is specified in the Appendix, Lemma 11, and is of either sign.

effect of roundabout production is neutralized.[23] Because $\alpha_1 = 1$ by assumption, it follows from (47) that $H(t^{R0}) = t^{het} - t^{R0} > 0$. It follows from the intermediate value theorem that there exists a tariff t_1^* with $t^{R0} < t_1^* < t^{het}$ at which $H(t_1^*) = 0$. By construction, this optimal tariff is a fixed point of (37) with $t_1^* < t^{het}$.

Part (b) deals with the opposite case where there is no roundabout production. In that case, the term $R(t_1)$ disappears from (47) so we only need to work with the term $M(t_1)$ reflecting the monopoly distortion between sectors. It turns out that $A(t_1) > 0$ is guaranteed in this case, so the sign of M is determined by the sign of D. Condition (42) used in part (b)(i) ensures that the relative distortion in the traded sector sufficiently exceeds that in the nontraded sector so that $D(t_1) > 0$ for $t_1 \in [1, t^{het}]$. It follows that $H(t^{het}) = -t^{het}\alpha_2 M(t^{het}) < 0$. We further show that there exists a sufficiently low tariff $t^{D0} < 1$ at which $D(t^{D0}) = 0$, so the monopoly distortion between sectors is neutralized.[24] In that case, $H(t^{D0}) = t^{het} - t^{D0} > 0$. It follows once again from the intermediate value theorem that there exists an optimal tariff t_1^*, with $t_1^{D0} < t_1^* < t^{het}$.

On the other hand, if the nontraded sector is sufficiently more distorted than the traded sector, with $\frac{\sigma_2}{(\sigma_2-1)} \geq t^{het}\frac{\sigma_1}{(\sigma_1-1)}$, then we have the reverse outcome with $D(t_1^*) < 0$ and $t_1^* > t^{het}$. In this case we find that $D(t^{het}) < 0$ and $M(t^{het}) < 0$, so $H(t^{het}) = -t^{het}\alpha_2 M(t^{het}) > 0$. The negative sign for the monopoly distortion indicates that resources should be shifted away from sector 1. We further show that there is a high enough tariff $t_1'' > t^{het}$ at which $M(t_1'') = 0$, so the monopoly distortion is neutralized.[25] Then we find from (47) with $\gamma_1 = 0$ that $H(t_1'') = t^{het} - t_1'' < 0$. It follows from the intermediate value theorem that there exists an optimal tariff t_1^*, now with $t^{het} < t_1^* < t_1''$. So the general conclusion is that without roundabout production, the tariff on final goods can be greater or less than that found in a one-sector model, depending on the relative monopoly distortion across sectors.

In part (c) we allow for two sectors and roundabout production, and so we need to ensure $A(t_1) > 0$. We establish that $A(t_1) > 0$ for $t_1 > t_1'$, where $t_1' < 1$ is an import subsidy specified in

[23]This result is obtained in Appendix E.1 because for $t_1 < 1$ then $\Lambda_1 > 1$, and so we can prove that the term in brackets in (40) equals zero at a point $t^{R0} < 1$.

[24]This result is obtained in Appendix E.3 because for $t_1 < 1$ then $\frac{\Lambda_1(1-\tilde{\gamma}_1)}{1-\tilde{\gamma}_1\Lambda_1} > 1$, and so under condition (42) we can prove that the terms in (35) sum to zero at a point $t^{D0} < 1$.

[25]This result is obtained in Appendix E.4 because we prove that there exists a high tariff $t_1'' > t^{het}$ at which $\mathcal{E}_m - [(t_1'' - 1)/t_1'']\theta_1 = 0$ in (38), and therefore $M(t_1'') = 0$.

the proof, under the sufficient conditions (43) and (44): the former is an upper-bound on γ_1 and the latter is an upper-bound on α_2 (but also depending on γ_1). There are two further conditions in part (c), and these are used to establish the sign of $H(t_1)$ at two tariff values chosen like in the proof of part (a): namely, t^{R0} and t^{het}, which give the values

$$\begin{aligned} H(t^{R0}) &= (t^{het} - t^{R0}) - t^{R0}\alpha_2 M(t^{R0}), && (48) \\ H(t^{het}) &= -t^{het}\left[\gamma_1 R(t^{het}) + \alpha_2 M(t^{het})\right]. && (49) \end{aligned}$$

We can establish that $M(t^{R0}) < 0$ provided that conditions (43) and (44) hold so that $A(t^{R0}) > 0$ (see Appendix E.5), and it follows that $H(t^{R0}) > 0$. Then the remaining conditions (45) and (46) in part (c) are used to show that $H(t^{het}) < 0$ in (49), because $\gamma_1 R(t^{het}) > -\alpha_2 M(t^{het})$. We know that $R(t^{het}) > 0$ and we are allowing the relative monopoly distortion to be of either sign, so in the case where $M(t^{het}) < 0$ then we see that (49) requires a sufficient amount of roundabout production, i.e., $\gamma_1 > -\alpha_2 M(t^{het})/R(t^{het}) > 0$. In that case we can apply the intermediate value theorem one last time to obtain the optimal tariff t_1^* with $t^{R0} < t_1^* < t^{het}$. Condition (46) is an upper-bound on the effective distortion in sector 2 relative to sector 1, and it generalizes condition (42) in to now allow for roundabout production. We argue in the next section that the constraints (43)–(45) are satisfied for all countries in our sample, while the upper-bound in (46) is satisfied for most.

We conclude this section by noting that the optimal tariff can be negative, as we earlier suggested following (26) when η_{m1} is sufficiently large. Consider the limiting case as $\lambda_{d1} \to 0$, so that $\eta_{m1} \to +\infty$. For simplicity, let us focus on a one-sector economy, so that $\alpha_1 = 1$. In that case, we can take the limiting value of the fixed-point formula in (37) (see Appendix E.6) to show that

$$\lim_{\lambda_{d1} \to 0} \; t_1^* = \frac{\theta_1 \rho_1}{(\theta_1 - \rho_1)} < 1, \;\; \text{when} \;\; \alpha_1 = 1 \text{ and } \gamma_1 < 0. \tag{50}$$

Remarkably, we find that the optimal tariff in this limiting economy exactly equals the subsidy in (18) found by Demidova and Rodríguez-Clare (2009): that subsidy is needed to correct the externality arising in a model with imported differentiated goods, whereby each new foreign variety brings surplus that domestic buyers do not take account of in their spending. Because the share of domestic inputs in the economy is vanishingly small, it appears that the additional tariff of $t^{hom} = 1/\rho_1$ identified by Gros (1987) and used by Demidova and Rodríguez-Clare in conjunction with the subsidy to obtain t^{het} (see equation (19)) is not needed anymore, so we are left with just the subsidy as the optimal policy in this limiting case. By continuity, any economy with a sufficiently small domestic share will also have a negative optimal *ad valorem* tariff.

This is not the only example of a negative optimal tariff, however. In Caliendo, Feenstra, Romalis and Taylor (2020), we examine the conditions to ensure that the optimal tariff is negative in a model with two symmetric countries, where only one country is applying the tariff. We find that a negative optimal tariff applies in two cases: *Highly Linked Economies* that have high roundabout production (high γ_1) and are very open (low λ_{d1}); and *Remote Economies*, with a small traded sector and with $\lambda_{d1} \to 1$, so that the economy is nearly closed to trade due to high iceberg costs, as may occur for very distant countries. The Highly Linked Economies are very similar to the negative optimal tariff found in (50) (except that (50) holds for all $\gamma_1 > 0$, so it does not require a high amount of roundabout production). The Remote Economies are different, however, and apply at the other extreme of the domestic share when $\lambda_{d1} \to 1$.

6 Second-Best Tariffs Around the World

We now take the model to the data and solve for second-best optimal tariffs. We use data from EORA 26 (Lenzen et.al., 2012, 2013) for the year 2010 which contains information for the world economy. Before we compute the optimal tariffs we need to aggregate the data into a two-country two-sector world. We define the Tradable goods sectors as sectors 1 through 12 from the EORA classification and the Nontradable goods sectors as sectors 13 through 25 from EORA classification (sector 26 is re-exports). For each country in EORA we aggregate all variables into these two sectors, and then for each country we aggregate all the others into the rest of the world (RoW). EORA contains information for 189 countries, many of which are small economies. However, as a preliminary step, and in order to determine the reliability of the data, for each country in the sample we compute the total GDP documented in EORA relative to the value documented by the World Bank indicators. Some countries had GDP values in EORA that represented more than 2 times or less than half the GDP value documented by the World Bank. We excluded these countries from the sample. We also excluded countries in the sample for which we do not have 2010 tariff data, which are needed to calibrate the model. As a result, we end up with a list of 164 countries in our sample (and for each country the RoW).[26]

The requirements to take the model to the data for each country are the following: the values of the finished goods produced in each sector Y_s, the domestic expenditure shares λ_{d1}, the labor share in each sector in our model, $1 - \gamma_s$, which more generally should be measured as the share of value added (i.e., payments to labor and capital) in the the variable costs of production. We also need information on the elasticities of substitution in each sector σ_s, and the Pareto share parameter θ_s. We first describe how we obtain these variables and then describe how we obtain the elasticities.

When taking the model to the data we need to deal with three issues. First, in order to measure the share of value added in production one cannot take the shares of industry revenue that go to value added directly from the data since the share of value added also includes profits (or "operating surplus"). Second, total intermediate goods includes purchases from your own sector and other sectors, but our model only has purchases of intermediates from your own sector. Third, our model assumes a sector with no trade, but the service industries in EORA have some trade.

For the first issue, we compute the share of intermediate goods in the cost of production for the Tradable and the Nontradable sectors as the intermediate goods purchased from the same sector divided by the sum of the compensation of employees, the consumption of fixed capital and the total intermediate goods purchased. So the latter three terms are used to measure the costs of production (and in particular they exclude "operating surplus", or profits). For the second issue, we include only the intermediate goods purchased from the *same sector* in roundabout production, but we cannot simply ignore the off-diagonal elements of the input-output matrix. By including *all* intermediate purchases in the cost of production, we are essentially attributing the expenditure on goods from other sectors into value added. Note that this is a conservative approach to measuring roundabout production, since it increases the share of value added in production and therefore reduces the share of intermediates. For the final issue, we excluded international trade in services from our calculations, so services is our Nontradable sector.

[26] See Appendix G, Table 3 for the full set of countries.

We measure the value of final goods produced in sector 1, Y_1, as the sum of the total domestic purchases plus total imports. In the case of sector 2 we have that Y_2 is equal to the total domestic purchases. We calculate the domestic expenditure share λ_{d1} as the share of domestic purchases over Y_1. It follows that $\lambda_{m1} = 1-\lambda_{d1}$. Then, given the level of tariffs, we can measure $\Lambda_1 \equiv \lambda_{d1}+(\lambda_{m1}/t)$. Given estimates of σ_s and the definitions in (3), we solve for total value added as

$$wL = \left((1-\gamma_1) + \frac{1}{\sigma_1 - 1}\right)\rho_1 \Lambda_1 Y_1 + \left((1-\gamma_2) + \frac{1}{\sigma_2 - 1}\right)\rho_2 Y_2, \tag{51}$$

and finally, the share of final goods in consumption is obtained using

$$\alpha_1 = \frac{(1-\tilde{\gamma}_1 \Lambda_1)\, Y_1}{wL + (1-\Lambda_1)\, Y_1}. \tag{52}$$

In order to obtain estimates for the elasticity of substitution and the Pareto parameter we use the estimates from Caliendo and Parro (2015). They show that by triple differencing the gravity equation one can identify the elasticities using tariff policy variation. In the context of our model the elasticity that is estimated is given by $1-\sigma_s\, \theta_s/(\sigma_s-1)$, and those values are reported in the first column of Table 1. In order to separately identify θ_s and σ_s we rely on estimates from the literature to obtain $\theta_s/(\sigma_s-1)$. The two most cited studies to deal with this issue are Chaney (2008) and Eaton, Kortum, and Kramarz (2011). Chaney finds that $\theta_s/(\sigma_s-1) = 2$ from U.S. sales data, while Eaton, Kortum, and Kramarz (2011, p. 1472) find an initial estimate of $\theta_s/\left(\sigma_s-1\right) = 1.75$ using French data on exporting firms. We rely on the latter estimate and apply it to the first column of Table 1 to obtain values for σ_s of 5.8, 8.3, and 3.7, respectively, for Agriculture and Fishing, Mining and Quarrying, and Manufacturing.[27] Gervais and Jensen (2019) find that services have an elasticity of substitution that is three-quarters the size of the elasticity in manufacturing (though they obtain rather high values for both elasticities using accounting data).[28] We follow them by setting $\sigma_2 = 0.75 \times 3.7 = 2.8$ for services, our Nontradable sector. Finally, we take a weighted average of the elasticities across the Tradable sector using the global shares of output shown in the final column of Table 1, obtaining $\sigma_1 = 4.5$. We therefore have σ_1 for Tradable goods considerably

[27] These elasticities are slightly revised from our working papers, CFRT (2020, 2021).
[28] This estimate of 0.75 comes from their working paper, Gervais and Jensen (2013).

Table 1: Elasticities

Sector(s)	$\frac{\sigma_s\theta_s}{\sigma_s-1}-1$	θ_s	σ_s	Global Share
Agriculture and Fishing	9.11	8.4	5.8	0.16
Mining and Quarrying	13.53	12.8	8.3	0.09
Manufacturing Sectors	5.55	4.8	3.7	0.75
Total Tradable Sector (above 3 sectors)	—	6.1	4.5	1
Total Nontradable Sector (all services)	—	3.1	2.8	—

Table 2: Distribution of parameters by countries and sectors

Statistic	Tradable	Nontradable
α_s $(p10)$	0.21	0.60
α_s $(median)$	0.25	0.75
α_s $(p90)$	0.40	0.79
$(1-\gamma_s)$ $(p10)$	0.34	0.75
$(1-\gamma_s)$ $(median)$	0.45	0.84
$(1-\gamma_s)$ $(p90)$	0.57	0.89
$\tilde{\sigma}_s = 1+(1-\gamma_s)(\sigma_s-1)$ $(p10)$	2.20	2.35
$\tilde{\sigma}_s = 1+(1-\gamma_s)(\sigma_s-1)$ $(median)$	2.57	2.51
$\tilde{\sigma}_s = 1+(1-\gamma_s)(\sigma_s-1)$ $(p90)$	3.01	2.60

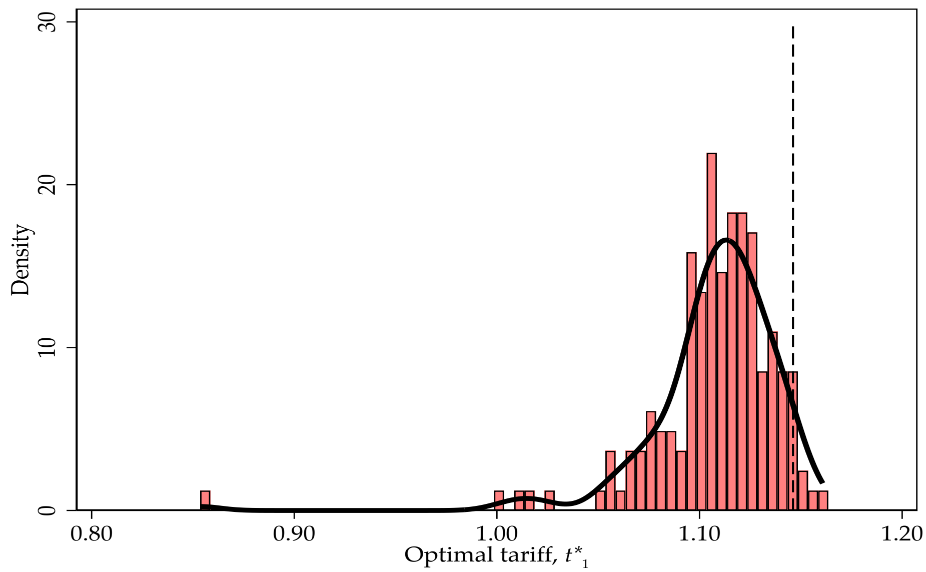
higher than σ_2 for Nontradable services, generating higher markups in the latter sector.

Table 2 reports the shares of industry final consumption, α_s as well as the share of industries revenue that go to value-added, $(1-\gamma_s)$. As expected, the share of expenditure on final goods in the Tradable sector is lower than in the Nontradables sector in our sample. The median share is 25% for Tradables (α_1) and 75% for Nontradables (α_2). We can see that the value added share for Tradables varies across countries from 34% at the 10th percentile to 57% at the 90th. Also reported is the effective elasticity $\tilde{\sigma}_s \equiv 1+(1-\gamma_s)(\sigma_s-1)$ in each sector. We find that the median effective elasticity in Tradables (2.57) is quite close to the median effective elasticity in Nontradables (2.51), so the effective monopoly distortion in the two sectors has similar median but still differs across countries.

To compute the optimal tariffs we then solve numerically the system of equations of the small open economy model using the "hat-algebra" method for large changes. We then verified that the solution coincides with the exact solution to the optimal tariff using our formula $H(t_1^*) = 0$ in (47).[29] Figure 2 reports the distribution of optimal tariffs for the 164 countries in our sample. The

[29]See Appendix F, where Figure 5 presents a scatter plot between the numerical solution from the hat-algebra and the exact solution, which are closely aligned. Table 3 in the Appendix includes the optimal tariff for each country in our sample along with the parameter values.

Figure 2: Distribution of optimal second-best tariffs (exact solution)



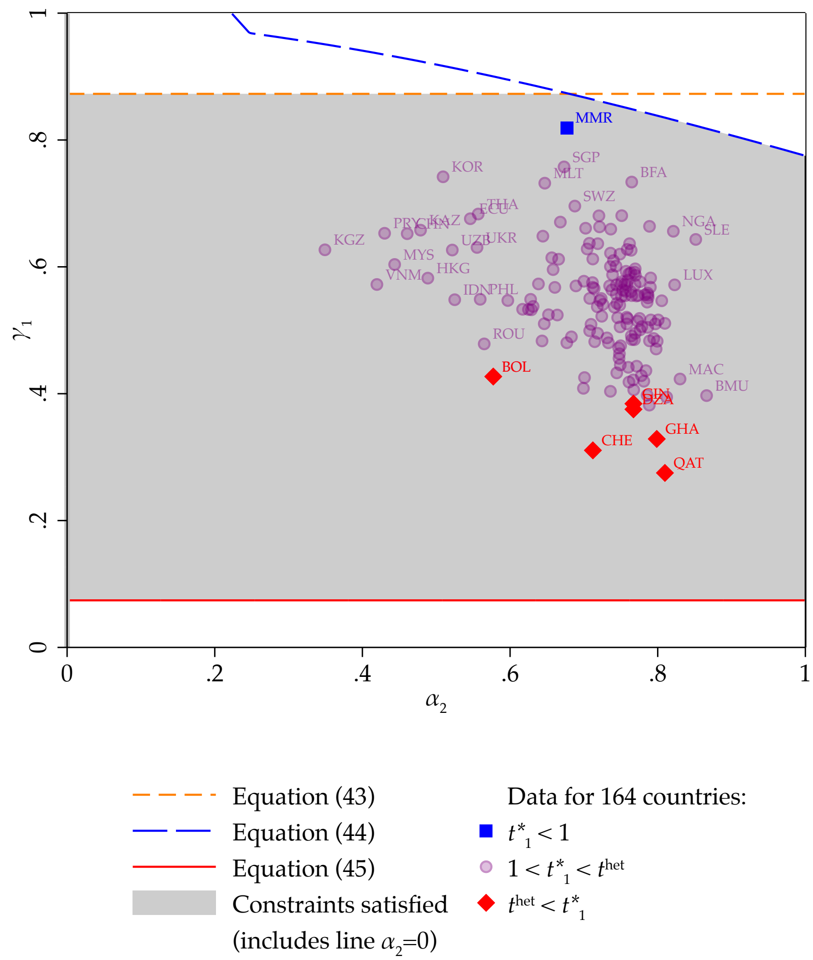
vertical dashed line represents the tariff value of $t^{het} = \frac{\theta_1 \rho_1}{(\theta_1 - \rho_1)} = 1.146$ or an *ad valorem* value of 14.6%. As we can see, almost all countries in our sample have an optimal tariff that is below t^{het}, and the median *ad valorem* optimal tariff is 11%, with much variation across countries.

The parameters in Table 2 can be used to illustrate how our optimal tariffs from the quantitative model accord with the predictions of Theorem 1. Each scatterplot dot in Figure 3 corresponds to the values of α_2 and γ_1 for the 164 countries in our sample, and we graph the constraints (43)–(45) from Theorem 1. We see that these constraints are satisfied for all countries in our sample.[30]

The final constraint that should be checked in Theorem 1 concerns the upper-bound on the effective distortion in Nontradables as compared with Tradables, given by (46). This constraint depends on γ_2, so it cannot be graphed here, but rather needs to be checked on a country-by-country basis. There are six countries that are highlighted in the lower-portion of Figure 2 with relatively low values of roundabout production γ_1: these countries all have $t_1^* > t^{het}$ and they violate the

[30]There is one country that is omitted from our sample that lies on the edge of a constraint, and that is Kuwait. However, we found that γ_1 for Kuwait is very sensitive to how we measure value-added: i.e., whether is consists of payments to labor and capital (as followed in this paper), or alternatively, whether it consists of all categories of value-added included in EORA (as followed in our working papers CFRT, 2020, 2021), which in addition to payments to labor and capital also includes operating surplus (i.e., profits), taxes paid, and a miscellaneous category of "mixed income". We did not observe this sensitivity in γ_1 depending on how value-added is measured for other countries, and for this reason, we have excluded Kuwait from our sample.

Figure 3: Parameter restrictions



constraint in (46).[31] In other words, these six countries have high enough values for the effective distortion in Nontradables that the (modest) amount of roundabout production in the Tradable sector is not enough to lead to $t_1^* < t^{het}$, contrary to what we find for other countries.

There is also one country highlighted at the top of Figure 3 and that is Myanmar (MMR), which has $t_1^* = 0.85$ so the optimal *ad valorem* tariff is -15%. Myanmar (formerly Burma) is an extremely closed country, and its domestic share evaluated at the optimal tariff is t_1^* is $\lambda_1^* = 0.998$.

[31] In addition, there are seven other countries – generally appearing in the lower portion of Figure 3 – that violate (46), which is a sufficient but not necessary conditions to have $t_1^* < t^{het}$. The median value of κ_0 in our sample is -0.184, which is not too different from the value $-1/\theta_1 = -0.164$ appearing in constraint (42) in Theorem 1. But the presence of $\kappa_1 = (t^{het} - \tilde{\gamma}_1)/(1 - \tilde{\gamma}_1)$ in (46), with a median value of 1.26, makes this a notably weaker constraint due to the presence of roundabout production than (42.

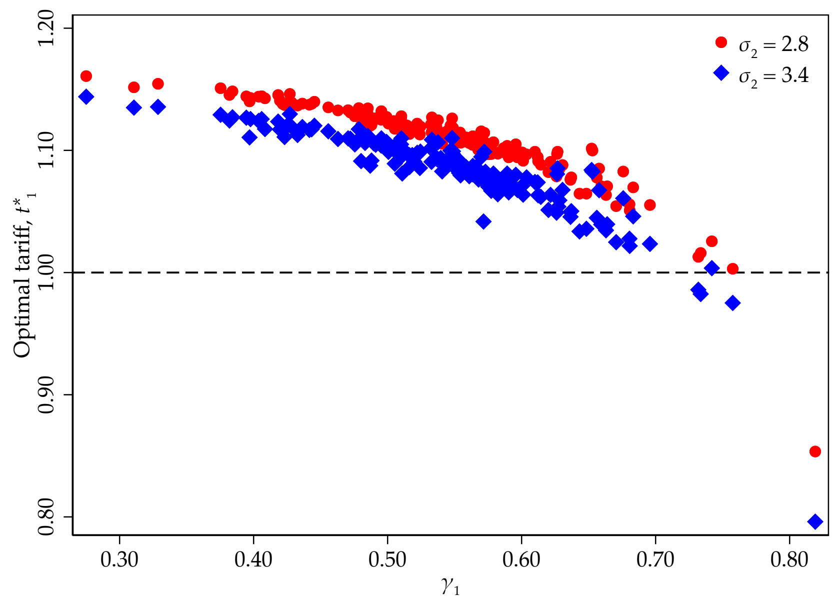
Just below Myanmar are two other labeled countries that have optimal *ad valorem* tariffs very close to zero, i.e., $t_1^* \in (1, 1.02)$, and are very open: Singapore (SGP, $\lambda_1^* = 0.27$) and Malta (MLT, $\lambda_1^* = 0.48$). Burkina Faso (BFA) is also labeled at the top of Figure 3 with $t_1^* \in (1, 1.02)$, and it is relatively closed with a domestic share $\lambda_1^* = 0.76$, above the median of 0.70. We will show in a sensitivity analysis below that in an alternative calibration where we modestly increase the value for σ_2 to a still plausible value, which acts to reduce the distortion in the Nontradable sector, then Singapore, Malta, Burkina Faso can also then have negative optimal tariffs.

To gain further insight, we performed a variance decomposition in the spirit of Eaton, Kortum and Kramarz (2004) to determine the contribution to the variance of the optimal tariff coming from roundabout production in the numerator of (37) versus the relative distortions across sectors in the denominator. Specifically, we write the numerator as $\ln[t^{het}(1 - \gamma_1 R(t_1^*))]$ and the denominator as $\ln[1 + \alpha_2 M(t_1^*)]$. Using each of these as dependent variables, we run a regression with $\ln t_1^*$ on the right. By construction, the two regression coefficients sum to unity, and they indicate the fraction of the variance in $\ln t_1^*$ explained by the numerator and the denominator of the fixed-point formula. We find that roundabout production explains 47% while the relative distortions across sectors explains 53% of the variation. Thus, in our calibrated model, roundabout production and the relative monopoly distortion are about equally important in explaining the variation in the optimal tariffs.

Recall that in our calibration of elasticities, we have relied on Gervais and Jensen (2013, 2019) who found that σ_2 for services is three-quarters that of σ_1 in manufacturing. That gave us the value $\sigma_2 = 2.8 = 3.7 \times 0.75$ used in our benchmark analysis. Because we also aggregate the Tradable sector over Manufacturing, Mining and Agriculture (see Table 1), we obtain a higher value for $\sigma_1 = 4.5$ in Tradables overall than in Manufacturing, so the elasticity used in Nontradables is considerably lower than that used in Tradables. As an alternative sensitivity check, we make a different assumption: we apply the factor of 0.75 from Gervais and Jensen to the elasticity used in the Tradable sector overall, obtaining the higher value of $\sigma_2 = 3.4 = 4.5 \times 0.75$ for the Nontradable sector.

In Figure 4 we graph the optimal tariff against γ_1 for our benchmark calibration (with $\sigma_2 = 0.28$) and for this alternative sensitivity check (with $\sigma_2 = 0.34$). In both cases, we see that there is a

Figure 4: Optimal tariff t_1^* versus roundabout parameter γ_1



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remarkably strong nonlinear relationship between t_1^* and γ_1. Raising σ_2 lowers all the optimal tariffs. With $\sigma_2 = 0.34$ we find that Myanmar is joined by Burkina Faso, Malta and Singapore in having negative optimal tariffs, with South Korea (KOR) now having an optimal *ad valorem* tariff very close to zero. This set of countries illustrate the theoretical result mentioned at the end of the previous section: negative optimal tariffs are likely to be found for both *Highly Linked* and *Remote* economies, but in all cases we find empirically that having a high value for γ_1 – indicating high roundabout production – is an essential feature.[32] Furthermore, by raising σ_2 we now find that there are *no* countries having a high optimal tariff, with $t_1^* > t^{het}$.

7 Conclusions

We began by asking whether modern trade theory has anything new to say about arguments for protecting the traded sector. It does, but the answer is nuanced. Gros (1987) showed that the Krugman model of monopolistic competition calls for a positive optimal tariff even for a small

[32] As mentioned in note 4, in CFRT (2021), we analyze a 186-country, 15-sector quantitative model for 2010 with a general input-output structure, and we find a negative optimal tariff for five countries, including Myanmar. In CFRT (2020), we analyze the same quantitative model for 1990, and we find a negative optimal tariff in ten countries: China, Hong Kong, India, Israel, Vietnam, and five more remote countries. Having a negative optimal tariff suggests that the welfare gains to these countries from unilateral tariff reductions from 1990 were of the first-order.

country. While we have explained that this tariff equalizes the monopoly markup on the price of domestic goods with the tariff distortion (i.e., one plus the *ad valorem* tariff) on the price of imports, other interpretations are possible. In particular, because of product differentiation in the Krugman model, the foreign demand curve for a home export variety is not infinitely elastic for a small country, but slopes downward. An import tariff – which is equivalent to an export tax by Lerner symmetry – reduces exports and therefore raises the export price, which is a terms of trade gain for the SOE applying the tariff. Even without imperfect competition, the presence of product variety on its own leads to a positive optimal tariff for a small country.[33]

The market structure in the SOE influences the optimal tariff, however. Demidova and Rodríguez-Clare (2009) found that the optimal tariff in a SOE with one sector and heterogeneous firms is lower than that obtained with homogeneous firms, so as to correct an externality in attracting foreign varieties. We have introduced a nontraded sector into the model, with roundabout production in both sectors. We find that there are strong reasons for the optimal tariff to be lower still, though this result is not guaranteed. With roundabout production, the idea of introducing a tariff distortion equal to the domestic markup breaks down: this policy would increase the price of the finished good that is bundled from the imported and domestic varieties, so that firms use too little of this finished good as compared to labor. To offset this distortion in the absence of any other policies, a *lower* value of the tariff is generally needed. The only exception to this rule occurs when the nontraded sector itself has a higher monopoly markup that the traded sector, which argues for a higher tariff to shift resources towards the nontraded good. For the vast majority of countries in our sample, the incentive to lower the tariff (to offset the distortion in the price of the finished good) is greater than the incentive to raise the tariff (when the nontraded sector is more distorted), and we find that the optimal tariff is lowered, and can be negative.

Our results stand in contrast to another literature that to some extent argues in favor of import protection. Specifically, this is the *firm-delocation* literature that combines a monopolistically competitive traded sector with a competitive *traded* outside good (see, e.g., Venables, 1987; Melitz and Ottaviano, 2008, section 4; Bagwell and Lee, 2020). The traded numeraire good pins down relative wages between countries, so the country applying tariffs is "small" in the sense that its

[33] This point is made by Caliendo and Parro (2022) in the context of a small country in the Eaton-Kortum model.

wages do not respond to its tariff. In this literature, encouraging entry into traded goods requires positive import tariffs. Essentially, the ability to attract firms into the home country takes the place of a conventional terms-of-trade motive for tariffs, so that the optimal tariff is positive even though wages are fixed. Of course, with multiple countries pursuing this motive for protection, there is ample scope for trade agreements to reduce the deadweight losses due to the tariffs (Ossa, 2011; Bagwell and Staiger, 2015).

The major differences between this class of models and our own are: (i) roundabout production, so that tariffs are applied on imported intermediate inputs rather than final goods; and (ii) the *nontraded* service sector, which does not fix relative wages between countries. Lerner symmetry holds in the traded sector, so that import tariffs are equivalent to export taxes and inhibit entry into that sector. That logic does not apply when the numeraire good is traded, which gives firm-delocation models a very different flavor: they act like partial equilibrium models because wages are fixed, and perhaps are most appropriate to narrowly targeted tariffs, whereas our results depend on Lerner symmetry, which is a general equilibrium property and depends on having broad tariffs applied to the traded sector. Determining the most appropriate range of applications for each class of models, and therefore the policy implications, is one important area for further research.

A second area for research is to investigate whether the optimal second-best tariff is low in other models beyond those we have investigated here. As we noted in section 5.1, in the presence of roundabout production the positive impact of an import tariff on the home wage can be reversed: evaluated at free trade, a rise in the tariff can lead to a fall in the home wage, and this is more likely under heterogeneous firms than with homogeneous firms. That negative terms of trade impact is crucial to obtaining an optimal tariff that is negative, and the question is whether this result extends outside the monopolistic competition framework. Consider, for example, the perfect competition Armington and Eaton-Kortum models. In the absence of intermediate inputs, Caliendo and Feenstra (2022) have shown that there is a formula for the optimal tariff that depends critically on the wage impact of the tariff, and this formula holds under monopolistic competition and these perfect competition models. What has not been investigated is whether the terms of trade impact itself can become negative in these competitive models due to input-output linkages.

Extending this question further, consider the perfect competition model with external economies of scale as analyzed by Bartelme, Costinot, Donaldson and Rodriguez-Clare (2019). They have shown that the optimal policy in a small economy is to have production subsidies to internalize the external economies of scale and export taxes to internalize the terms-of-trade externalities. Furthermore, they show that this policy combination continues to hold with intermediate goods and an input-output structure. When production subsidies are not feasible, so that we are in a second-best setting, other questions for research are whether the terms of trade impact of a tariff/tax is reduced due to input-output linkages, and therefore whether the second best tariff/tax is lower than in the first-best.

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ONLINE APPENDIX

A Two-Sector Small Open Economy Model

We focus on a two country model with two sectors $s = 1, 2$, and roundabout production in both sectors. The home county is a small open economy (SOE) that applies an import tariff t_1 in sector 1, while sector 2 is nontraded. In the foreign country, for notational simplicity we assume a single traded sector denoted by $s = 1$ with no roundabout production and no import tariff, but these assumptions are not important: what matters for the foreign country is that its price index and mass of goods are held fixed, as will be described below.

A.1 Description of the Economy

The structure of the home economy is illustrated in Figure 1. Firms in sector 1 at home can source differentiated inputs from home or abroad, and the CES production functions over the differentiated inputs purchased from home (Q_{d1}) and imported (Q_{m1}):

$$Q_{d1} \equiv \left(N_1^e \int_{\varphi_{d1}}^{\infty} q_{d1}(\varphi)^{\frac{\sigma_1 - 1}{\sigma_1}} g_1(\varphi) d\varphi \right)^{\frac{\sigma_1}{\sigma_1 - 1}} \quad \text{and} \quad Q_{m1} \equiv \left(N_1^{e*} \int_{\varphi_{x1}^*}^{\infty} q_{x1}^*(\varphi)^{\frac{\sigma_1 - 1}{\sigma_1}} g_1(\varphi) d\varphi \right)^{\frac{\sigma_1}{\sigma_1 - 1}}, \quad (53)$$

which gives the combined home output of the bundled good:

$$Q_1 \equiv \left(Q_{d1}^{\frac{\sigma_1 - 1}{\sigma_1}} + Q_{m1}^{\frac{\sigma_1 - 1}{\sigma_1}} \right)^{\frac{\sigma_1}{\sigma_1 - 1}}.$$

In these expressions, N_1^e is the mass of entrants at home that will be derived below, while N_1^{e*} is the mass of entrants abroad that is exogenous. Firms in either country draw productivity from a Pareto distribution $g_1(\varphi)$, and then $q_{d1}(\varphi)$ and $q_{x1}^*(\varphi)$ are the respective sales of home and foreign firms to the home country. The productivities $\varphi \geq \varphi_{d1}$ and $\varphi \geq \varphi_{x1}$ are needed for a home firm to sell domestically and export, while $\varphi \geq \varphi_{x1}^*$ is needed for the foreign firms to export, as will be described below. In sector 2 the notation is similar, except that there are no foreign exports so that $Q_2 \equiv Q_{d2}$.

The CES price indexes over the differentiated inputs purchased domestically and imported are

$$P_{d1} = \left(N_1^e \int_{\varphi_{d1}}^{\infty} p_{d1}(\varphi)^{1-\sigma_1} g_1(\varphi)\, d\varphi \right)^{\frac{1}{1-\sigma_1}} \text{ and } P_{m1} = \left(N_1^{*e} \int_{\varphi_{x1}^*}^{\infty} p_{x1}^*(\varphi)^{1-\sigma_1} g_1(\varphi)\, d\varphi \right)^{\frac{1}{1-\sigma_1}}, \quad (54)$$

where p_{ds} and p_{x1}^* are the domestic and foreign export prices, with the latter inclusive of all tariffs and transport costs. Then the overall price index is

$$P_1 = \left(P_{d1}^{1-\sigma_1} + P_{m1}^{1-\sigma_1} \right)^{\frac{1}{1-\sigma_1}}.$$

In sector 2 the notation is similar, except that with no foreign exports we have $P_2 = P_{d2}$. The mass of input varieties that sector 1 firms sell at home and export are

$$N_{k1} = N_1^e[1 - G_1(\varphi_{k1})] = N_1^e \varphi_{k1}^{-\theta_1}, \text{ for } k = d, x, \quad (55)$$

using the Pareto distribution $G_1(\varphi) = 1 - \varphi^{-\theta_1}$ with $\varphi \geq 1$. Notice that the entry of firms N_{k1} appearing in (53) and (54) can be converted into the mass of varieties by multiplying and dividing by $[1 - G_1(\varphi_{k1})]$, in which case the unconditional densities $g_1(\varphi)$ become conditional densities $g_1(\varphi)/[1 - G_1(\varphi_{k1})]$. Likewise, in sector 2 we have $N_{d2} = N_2^e[1 - G_2(\varphi_{d2})] = N_2^e \varphi_{d2}^{-\theta_2}$.

For home as a SOE, the foreign price index P_1^* is exogenous. The condition $N_{x1}^* = N_1^{e*} {\varphi_{x1}^*}^{-\theta_1}$ holds for the foreign varieties sold at home, analogously to (55). The small home country means that we take N_1^{e*} is exogenous, but the mass of foreign exported varieties N_{x1}^* is endogenous because the foreign cutoff φ_{x1}^* is also endogenous, as described below.

The total value of production of the finished good in sector s is $Y_s = P_s Q_s$, and the CES demand for intermediates of variety φ sold at home and imported are

$$q_{ds}(\varphi) = \left(\frac{p_{ds}(\varphi)}{P_s} \right)^{-\sigma_s} \frac{Y_s}{P_s}, \; s = 1, 2, \quad (56)$$

$$\text{and } q_{m1}(\varphi) = \left(\frac{p_{x1}^*(\varphi)}{P_1} \right)^{-\sigma_s} \frac{Y_1}{P_1}. \quad (57)$$

Denoting the home export price by p_{x1}, then demand for exported varieties in sector 1 is

$$q_{x1}(\varphi) = \left(\frac{p_{x1}(\varphi)}{P_1^*}\right)^{-\sigma_s} \frac{Y_1^*}{P_1^*}. \tag{58}$$

A home firm with productivity φ has marginal costs c_s/φ in sector s, with the input cost index c_s given by (1). We assume that fixed costs of the firm require only labor and are denoted by f_s and f_s^e. The foreign costs of an exporting firm are c_s^*/φ where c_s^* is exogenous by normalizing the foreign wage at unity $w^* = 1$ and also treating the foreign price index P_1^* as exogenous. Fixed labor costs abroad are f_1^* and f_1^{e*}.

The profits from supplying differentiated inputs at home and exporting are

$$\pi_{ks}(\varphi) = \max_{p_{ks}(\varphi)\geq 0} \left\{ p_{ks}(\varphi) q_{ks}(\varphi) - \frac{c_{ks}}{\varphi}\, \tau_{ks}\, q_{ks}(\varphi) - w f_{ks} \right\},\ k = d, x, \tag{59}$$

where we ignore the export equation $\pi_{x2}(\varphi)$ in sector 2 because there are no exports, and τ_{ks} are iceberg trade costs with $\tau_{ds} \equiv 1$, $s = 1, 2$ and $\tau_{x1} \geq 1$. To evaluate the foreign profits from exporting, we divide the the c.i.f. value of imports – including cost, freight charges and the markup – by the tariff. Then the profit-maximization problem for the foreign exporter is

$$\pi_{x1}^*(\varphi) = \max_{p_{x1}^*(\varphi)\geq 0} \left\{ \frac{p_{x1}^*(\varphi)}{t_1} q_{x1}^*(\varphi) - \frac{c_{x1}^*}{\varphi}\, \tau_{x1}^*\, q_{m1}(\varphi) - \, w^* f_{x1}^* \right\}, \tag{60}$$

where t_1 is one plus the home *ad valorem* tariff and $\tau_{x1}^* \geq 1$ are the iceberg trade costs. Notice that if divide out the tariff t_1 from this expression, we obtain

$$\pi_{x1}^*(\varphi) = \max_{p_{x1}^*(\varphi)\geq 0} \frac{1}{t_1} \left\{ p_{x1}^*(\varphi) q_{x1}^*(\varphi) - \frac{c_{x1}^*}{\varphi}\, \tau_{x1}^*\, t_1 q_{m1}(\varphi) - \, w^* f_{x1}^* t_1 \right\}. \tag{61}$$

In other words, when choosing the c.i.f. price p_{x1}^*, the foreign firm acts "as if" it is facing the tariff t_1 on its marginal costs and on its fixed costs.[34]

[34] Costinot and Rodríguez-Clare (2014, note 30) take two different approaches: applying tariffs to the marginal costs of exporters, so the tariffs act as "cost-shifters"; and in their Appendix, applying tariffs to the c.i.f price of exports, so the tariffs act as "demand-shifters". We use only the latter approach: see also the discussion in Caliendo, Feenstra, Romalis and Taylor (2020, Appendix A) and note 36.

The first-order conditions for profit maximization yield the optimal prices

$$p_{ds}(\varphi) = \left(\frac{\sigma_s}{\sigma_s - 1}\right)\frac{c_s}{\varphi} \text{ for } s = 1, 2, \tag{62}$$

$$p_{x1}(\varphi) = \left(\frac{\sigma_s}{\sigma_s - 1}\right)\frac{c_1\,\tau_{x1}}{\varphi} \text{ and } \frac{p^*_{x1}(\varphi)}{t_1} = \left(\frac{\sigma_s}{\sigma_s - 1}\right)\frac{c^*_1\,\tau^*_{x1}}{\varphi}. \tag{63}$$

Substituting these expressions into (56)–(58) to obtain the quantities and then back into profits, we can readily solve for the cutoff productivity at which profits are zero:

$$\pi_{ds}(\varphi_{ds}) = 0 \implies \varphi_{ds} = \left(\frac{\sigma_s}{\sigma_s - 1}\right)\left(\frac{\sigma_s w f_{ds}}{Y_s}\right)^{\frac{1}{(\sigma_s-1)}}\frac{c_s}{P_s},\ s = 1, 2 \tag{64}$$

$$\pi_{x1}(\varphi_{x1}) = 0 \implies \varphi_{x1} = \left(\frac{\sigma_1}{\sigma_1 - 1}\right)\left(\frac{\sigma_1 w f_{x1}}{Y^*_1}\right)^{\frac{1}{(\sigma_s-1)}}\frac{c_1\,\tau_{x1}}{P^*_1}, \tag{65}$$

$$\pi^*_{x1}(\varphi^*_{x1}) = 0 \implies \varphi^*_{x1} = \left(\frac{\sigma_1}{\sigma_1 - 1}\right)\left(\frac{\sigma_1 w^*\, f^*_{x1}\, t_1}{Y_1}\right)^{\frac{1}{(\sigma_s-1)}}\frac{c^*_1\,\tau^*_{x1} t_1}{P_1}. \tag{66}$$

We follow Melitz (2003) in defining the average productivity as

$$\bar{\varphi}_{ks} \equiv \left(\int_{\varphi_{ks}}^{\infty} \varphi^{\sigma_s-1}\frac{g_s(\varphi)}{[1 - G_s(\varphi_{ks})]}d\varphi\right)^{\frac{1}{\sigma_s-1}} = K_s\varphi_{ks}, \text{with } K_s \equiv \left(\frac{\theta_s}{\theta_s - \sigma_s + 1}\right)^{\frac{1}{\sigma_s-1}}, \tag{67}$$

where $k = d, x$ for $s = 1$ and $k = d$ for $s = 2$, and the constant K_s is obtained by computing the integral using the Pareto distribution. We define the average foreign export productivity $\bar{\varphi}^*_{x1} = K_1\varphi^*_{x1}$ analogously.

We can substitute the quantities from (56)–(57) into (53) to obtain the output of the finished good in sector 1:

$$Q_1 = \left(\frac{\sigma_1}{\sigma_1 - 1}\right)^{-\sigma_1}\left(\frac{Y_1}{P_1^{1-\sigma_1}}\right)\left[N^e_1\varphi_{d1}^{-\theta_1}\left(\frac{c_1}{\bar{\varphi}_{d1}}\right)^{1-\sigma_1} + N^{e*}_1\varphi_{x1}^{*-\theta_1}\left(\frac{c^*_1\,\tau^*_{x1}\,t_1}{\bar{\varphi}^*_{x1}}\right)^{1-\sigma_1}\right]^{\frac{\sigma_1}{\sigma_1-1}}, \tag{68}$$

Likewise, we use the prices from (62) and (63) in (54) to obtain an expression for P_1:

$$P_1 = \left(\varphi_{d1}^{-\theta_1}N^e_1\left(\frac{\sigma_1}{\sigma_1 - 1}\frac{c_1}{\bar{\varphi}_{d1}}\right)^{1-\sigma_1} + \varphi_{x1}^{*-\theta_1}N^{e*}_1\left(\frac{\sigma_1}{\sigma_1 - 1}\frac{c^*_1\,\tau^*_{x1}\,t_1}{\bar{\varphi}^*_{x1}}\right)^{1-\sigma_1}\right)^{\frac{1}{1-\sigma_1}}. \tag{69}$$

We can multiply this by (68) to obtain a preliminary expression for the value of production of the finished goods $Y_1 \equiv P_1 Q_1$:

$$Y_1 = K_1^{\sigma_s - 1} \left(\frac{\sigma_1}{\sigma_s - 1} \right)^{1-\sigma_1} \left(\frac{Y_1}{P_1^{1-\sigma_s}} \right) \left[N_1^e \varphi_{d1}^{-\theta_1} \left(\frac{c_1}{\varphi_{d1}} \right)^{1-\sigma_s} + \ N_1^{e*} \varphi_{x1}^{*-\theta_1} \left(\frac{c_1^* \, \tau_{x1}^* \, t_1}{\varphi_{x1}^*} \right)^{1-\sigma_s} \right].$$

To simplify this expression, we can use (64) and (66) to obtain

$$\frac{Y_1}{P_1^{1-\sigma_1}} = \sigma_1 w f_{d1} \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c_1}{\varphi_{d1}} \right)^{\sigma_1 - 1} = \sigma_1 w^* f_{x1}^* t_1 \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c_1^* \, \tau_{x1}^* t_1}{\varphi_{x1}^*} \right)^{\sigma_1 - 1}.$$

and substituting above and also using (67) we obtain

$$Y_1 = K_1^{\sigma_1 - 1} \sigma_1 \left(N_1^e \varphi_{d1}^{-\theta_1} \, w \, f_{d1} + \ N_1^{e*} \varphi_{x1}^{*-\theta_1} \, w^* \, f_{x1}^* \, t_1 \right). \tag{70}$$

A similar expression can be obtained for sector 2: $Y_2 = K_2^{\sigma_2 - 1} \sigma_2 \left(N_2^e \varphi_{d2}^{-\theta_2} \, w \, f_{d2} \right).$

The value of finished output in each sector, Y_s, is sold to consumers and also back to domestic firms. That finished output is costlessly bundled from home and (for sector 1) imported differentiated inputs. Let λ_{k1} denote the share of home total expenditure in sector 1 on intermediate goods from the home (for $k = d$) and foreign (for $k = x$) countries. Using the above conditions we can obtain the following expressions for the expenditure shares:

$$\lambda_{d1} \ = \ \varphi_{d1}^{-\theta_1} \, N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1} \, \frac{c_1}{\bar{\varphi}_{d1} \, P_1} \right)^{1-\sigma_1} = \varphi_{d1}^{-\theta_1} \, N_1^e \left(\frac{\sigma_1 \, w \, f_{d1}}{Y_1} \right) \left(\frac{\theta_1}{\theta_1 + 1 - \sigma_1} \right), \tag{71}$$

$$\lambda_{x1} \ = \ \varphi_{x1}^{-\theta_1} \, N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1} \, \frac{c_1 \tau_{x1}}{\bar{\varphi}_{x1} \, P_1^*} \right)^{1-\sigma_1} = \varphi_{x1}^{-\theta_1} \, N_1^e \left(\frac{\sigma_1 \, w \, f_{x1}}{Y_1^*} \right) \left(\frac{\theta_1}{\theta_1 + 1 - \sigma_1} \right), \tag{72}$$

$$\lambda_{m1} \ = \ \varphi_{x1}^{*\,-\theta_1} \, N_1^{e*} \left(\frac{\sigma_1}{\sigma_1 - 1} \, \frac{c_1^* \, \tau_{x1}^* \, t_1}{\bar{\varphi}_{x1}^* \, P_1} \right)^{1-\sigma_1} = \varphi_{x1}^{*\,-\theta_1} \, N_1^{e*} \left(\frac{\sigma_1 w^* f_{x1}^* t_1}{Y_1} \right) \left(\frac{\theta_1}{\theta_1 + 1 - \sigma_1} \right). \tag{73}$$

In Appendix A.6 we discuss how the second equalities of (71)–(73) are obtained and interpreted.

The model is closed by making use of the market clearing condition described in the main text in (4), which in sector 2 is simply $Y_2 = \alpha_2(wL + B) + \tilde{\gamma}_2 Y_2$, together with trade balances. Home exports in sector 1 are $E_{x1} = \lambda_{x1} Y_1^*$, while duty-free imports are $E_{x1}^* = (\lambda_{m1} Y_1)/t_1$, so that trade balance requires

$$\lambda_{x1} Y_1^* = \frac{\lambda_{m1} Y_1}{t_1}. \tag{74}$$

Note that using (72) and (73), then trade balance (74) implies

$$\varphi_{x1}^{-\theta_1} N_1^e \, w \, f_{x1} = \varphi_{x1}^{*\,-\theta_1} N_1^{e*} \, w^* \, f_{x1}^*. \tag{75}$$

Again using (72) and (73) with home sales $E_{d1} = \lambda_{d1} Y_1$ and exports $E_{x1} = \lambda_{x1} Y_1^*$, we obtain an expression for total sales of intermediate inputs in sector 1:

$$E_{d1} + E_{x1} = \sum_{k=d,x} \varphi_{k1}^{-\theta_1} N_1^e \left(\frac{\theta_1 \sigma_1 \, w \, f_{k1}}{\theta_1 + 1 - \sigma_1} \right).$$

This equation is simplified by making use of free entry. Expected profits must equal the fixed costs of entry, so that:

$$\sum_{k=d,x} \int_{\varphi_{k1}}^{\infty} \pi_{k1}(\varphi) g_1(\varphi) d\varphi = w f_1^e. \tag{76}$$

To evaluate this integral we follow the approach of Melitz and Redding (2014), who note that CES demand implies that $\pi_{k1}(\varphi) + w f_{k1} = [\pi_{k1}(\varphi_{k1}) + w f_{k1}] \, (\varphi/\varphi_{k1})^{\sigma_1 - 1}$. It follows from (59) that $\pi_{k1}(\varphi) = \left[(\varphi/\varphi_{k1})^{\sigma_1 - 1} - 1 \right] w f_{k1}$, and so the above entry condition becomes:

$$J_1(\varphi_{d1}) f_{d1} + J_1(\varphi_{x1}) f_{x1} = f_1^e \quad \text{with} \quad J_1(\varphi_{k1}) \equiv \int_{\varphi_{k1}}^{\infty} \left[\left(\frac{\varphi}{\varphi_{k1}} \right)^{\sigma_1 - 1} - 1 \right] g_1(\varphi) d\varphi.$$

Completing the integral above using the Pareto distribution, we arrive at

$$\left(\frac{\sigma_1 - 1}{\theta_1 - \sigma_1 + 1} \right) \left(\varphi_{d1}^{-\theta_1} f_{d1} + \varphi_{x1}^{-\theta_1} f_{x1} \right) = f_1^e, \tag{77}$$

from which we can obtain an equation governing the mass of entrants N_s^e, namely

$$N_1^e = (E_{d1} + E_{x1}) \Big/ \left[w \, f_1^e \left(\frac{\theta_1 \, \sigma_1}{\sigma_1 - 1} \right) \right]. \tag{78}$$

In sector 2 the mass of entrants is governed by the same equation but without E_{x2} appearing

$$N_2^e = E_{d2} \Big/ \left[w\, f_2^e \left(\frac{\theta_1\, \sigma_2}{\sigma_2 - 1} \right) \right]. \tag{79}$$

The free entry condition for sector 2 is defined analogously to (76) but summing over domestic sales only, obtaining a condition that determines φ_{d2}:

$$J_2(\varphi_{d2}) f_{d2} = \left(\frac{\sigma_2 - 1}{\theta_2 - \sigma_2 + 1} \right) \varphi_{d2}^{-\theta_2}\, f_{d2} = f_2^e. \tag{80}$$

A.2 Domestic Production Share and $T(t_1)$

We now introduce the share of *production* of differentiated intermediate inputs that are sold domestically, which will be used many times in our derivations. The expenditure share on imported intermediate inputs is λ_{m1} in (73), so $\lambda_{m1} Y_1$ measures the value of imports inclusive of tariffs (and iceberg costs). We can instead evaluate imports at the net-of-tariff prices by dividing by t_1 obtaining $\lambda_{m1} Y_1 / t_1 = (1 - \lambda_{d1}) Y_1 / t_1$, which equals exports and can be summed with $\lambda_{d1} Y_1$ to obtain the total value of production of differentiated intermediate inputs. It follows that the *share* of production sold to domestic firms – or the domestic production share – is

$$\tilde{\lambda}_{d1} \equiv \frac{\lambda_{d1}}{\lambda_{d1} + \frac{(1 - \lambda_{d1})}{t_1}} = \frac{t_1 \lambda_{d1}}{1 + \lambda_{d1}(t_1 - 1)}. \tag{81}$$

When $t_1 = 1$ then $\tilde{\lambda}_{d1} = \lambda_{d1}$, but otherwise they differ. We claim that $\tilde{\lambda}_{d1}$ can be measured by

$$\tilde{\lambda}_{d1} = \frac{\varphi_{d1}^{-\theta_1} f_{d1}}{\varphi_{d1}^{-\theta_1} f_{d1} + \varphi_{x1}{}^{-\theta_1} f_{x1}}. \tag{82}$$

To show this, we first rewrite the domestic expenditure share λ_{d1} using (70), (71) and (75) as

$$\lambda_{d1} = \frac{\varphi_{d1}^{-\theta_1}\, f_{d1}}{\varphi_{d1}^{-\theta_1}\, f_{d1} + \varphi_{x1}^{-\theta_1}\, f_{x1} t_1}. \tag{83}$$

For the above two equations we obtain the relationship

$$t_1 = \frac{(1-\lambda_{d1})}{\lambda_{d1}} \frac{\tilde{\lambda}_{d1}}{(1-\tilde{\lambda}_{d1})}, \tag{84}$$

and as a result

$$t_1 - 1 = \frac{\tilde{\lambda}_{d1} - \lambda_{d1}}{\lambda_{d1}\left(1-\tilde{\lambda}_{d1}\right)}. \tag{85}$$

Multiplying both of these equations by λ_{d1}, adding unity to the last equation, and taking their ratio, we readily confirm (81), which establishes that (82) is a correct formula for the domestic production share.

We can use this production share to define a simple function of the tariff $T(t_1)$ given by

$$T(t_1) \equiv 1 - \tilde{\gamma}_1 + (t_1 - 1)\left(1-\tilde{\lambda}_{d1}\right). \tag{86}$$

Notice that $T(t_1) = 1 - \tilde{\gamma}_1$ in free trade (with $t_1 = 1$) and autarky ($t_1 \to +\infty$ so $\lambda_{d1} = \tilde{\lambda}_{d1} = 1$), but $T(t_1) > 1 - \tilde{\gamma}_1$ for $1 < t_1 < +\infty$. It follows that $T(t_1)$ is a $\cap$-shaped function of the tariff between these two points, which is the same shape as tariff revenue B. In fact, $T(t_1)$ and B have their critical points at the same tariff, as we show below.

In the main text we use Λ_1 to characterize entry into sector 1, but throughout the rest of the Appendix we mainly find it convenient to instead use the function $T(t_1)$. These two concepts are inversely related, which can be seen by using (84) and (85) to obtain

$$t_1 = \frac{\tilde{\lambda}_{d1}}{\lambda_{d1}} \frac{(1-\lambda_{d1})}{(1-\tilde{\lambda}_{d1})} = [(t_1-1)(1-\tilde{\lambda}_{d1}) + 1] \frac{(1-\lambda_{d1})}{(1-\tilde{\lambda}_{d1})}.$$

Using this expression and $T(t_1)$ from (86), with $\lambda_{m1} = 1 - \lambda_{d1}$ we can solve for

$$\Lambda_1 \equiv \lambda_{d1} + \frac{\lambda_{m1}}{t_1} = 1 - \frac{(t_1-1)[(T(t_1)+\tilde{\gamma}_1) - 1]}{(t_1-1)(T(t_1)+\tilde{\gamma}_1)} = \frac{1}{(T(t_1)+\tilde{\gamma}_1)}. \tag{87}$$

We see that Λ_1 and $T(t_1)$ are inversely related, as asserted. Since $1 - \Lambda_1 = \frac{T(t_1)-(1-\tilde{\gamma}_1)}{T(t_1)+\tilde{\gamma}_1}$ from the above equation, we can substitute this into (9) to obtain

$$B = \frac{\alpha_1 wL[T(t_1) - (1-\tilde{\gamma}_1)]}{[T(t_1) - (1-\tilde{\gamma}_1)]\alpha_2 + 1 - \tilde{\gamma}_1} = \frac{\alpha_1 wL}{\alpha_2 + \frac{1-\tilde{\gamma}_1}{[T(t_1)-(1-\tilde{\gamma}_1)]}}. \tag{88}$$

We see that B is monotonically increasing in $T(t_1)$, so they have their critical points at the same maximum-revenue tariff. Note that if we take $\alpha_1 = 1$ so we are in a one-sector model, then B and $T(t_1)$ are simple affine transformations of each other, given by

$$B = wL\left(\frac{T(t_1)}{1-\tilde{\gamma}_1} - 1\right).$$

A.3 Labor Allocation

We now derive expressions for labor market demand in sectors 1 and 2:

$$\begin{aligned}
L_1 &= N_1^e f_1^e + N_1^e f_{d1} \int_{\varphi_{d1}}^{\infty} g_1(\varphi)\, d\varphi + N_1^e f_{x1} \int_{\varphi_{x1}}^{\infty} g_1(\varphi)\, d\varphi \\
&\quad + (1-\gamma_1)(\sigma_1 - 1) N_1^e \sum_{k=d,x} \left[\int_{\varphi_{k1}}^{\infty} \frac{\pi_{k1}}{w}(\varphi)\, g_1(\varphi)\, d\varphi + f_{k1} \int_{\varphi_{k1}}^{\infty} g_1(\varphi)\, d\varphi \right], \\
L_2 &= N_2^e f_2^e + N_2^e f_{d2} \int_{\varphi_{d2}}^{\infty} g_1(\varphi)\, d\varphi \\
&\quad + (1-\gamma_2)(\sigma_2 - 1) N_2^e \left[\int_{\varphi_{d2}}^{\infty} \frac{\pi_{d2}}{w}(\varphi)\, g_1(\varphi)\, d\varphi + f_{d2} \int_{\varphi_{d2}}^{\infty} g_1(\varphi)\, d\varphi \right].
\end{aligned}$$

Using the free entry condition (76), we obtain

$$\frac{L_1}{N_1^e} = (1 + (1-\gamma_1)(\sigma_1 - 1)) \left(f_1^e + f_{d1}\varphi_{d1}^{-\theta_1} + f_{x1}\varphi_{x1}^{-\theta_1} \right), \tag{89}$$

$$\frac{L_2}{N_2^e} = (1 + (1-\gamma_2)(\sigma_2 - 1)) \left(f_2^e + f_{d2}\varphi_{d2}^{-\theta_2} \right). \tag{90}$$

Also using (77) and (80), entry into sectors 1 and 2 becomes

$$N_1^e = \frac{(\sigma_1 - 1)}{[1 + (1 - \gamma_1)(\sigma_1 - 1)]\,\theta_1} \frac{L_1}{f_1^e}, \tag{91}$$

$$N_2^e = \frac{(\sigma_2 - 1)}{[1 + (1 - \gamma_2)(\sigma_2 - 1)]\theta_2} \frac{L_2}{f_2^e}. \tag{92}$$

Combining the expressions, we obtain

$$\frac{L_1}{L_2} = \frac{N_1^e}{N_2^e} \frac{\frac{[1+(1-\gamma_1)(\sigma_1-1)]\theta_1 f_1^e}{(\sigma_1-1)}}{\frac{[1+(1-\gamma_2)(\sigma_2-1)]\theta_2 f_2^e}{(\sigma_2-1)}}. \tag{93}$$

To characterize the labor allocation across sectors, we need to use entry. We have already solved for Y_1 in (8) and the analogous expression for Y_2 is

$$Y_2 = \frac{\alpha_2}{1 - \tilde{\gamma}_2} I. \tag{94}$$

Use these results in (78) and (79) and recall that home sales are $E_{d1} = \lambda_{d1} Y_1$ and $E_{d2} = Y_2$ while exports are $E_{x1} = \lambda_{x1} Y_1^* = \lambda_{m1} Y_1 / t_1$. Substituting the resulting expressions into (93), labor allocation across sectors can be written as

$$\frac{L_1}{L_2} = \frac{\alpha_1}{\alpha_2} \frac{(1 - \tilde{\gamma}_1)\left(\lambda_{d1} + \frac{\lambda_{m1}}{t_1}\right)}{\left[1 - \tilde{\gamma}_1\left(\lambda_{d1} + \frac{\lambda_{m1}}{t_1}\right)\right]}. \tag{95}$$

The tariff formula (84) derived earlier can be used to simplify this expression for labor allocation. Using (84) in (95), we obtain

$$\frac{L_1}{L_2} = \frac{\alpha_1}{\alpha_2} \frac{(1 - \tilde{\gamma}_1)\left(\frac{1+(t_1-1)\lambda_{d1}}{t_1}\right)}{\left[1 - \tilde{\gamma}_1\left(\frac{1+(t_1-1)\lambda_{d1}}{t_1}\right)\right]} = \frac{\alpha_1}{\alpha_2} \frac{(1 - \tilde{\gamma}_1)\left((1 - \tilde{\lambda}_{d1})t_1 + \tilde{\lambda}_{d1}\right)^{-1}}{\left[1 - \tilde{\gamma}_1\left(\left(1 - \tilde{\lambda}_{d1}\right)t_1 + \tilde{\lambda}_{d1}\right)^{-1}\right]}.$$

Then we can also express the labor allocation as a fraction of total labor supply:

$$\frac{L_2}{L_1 + L_2} = \left(\frac{L_1}{L_2} + 1\right)^{-1} = \frac{(1 - \tilde{\lambda}_{d1})t_1 + \tilde{\lambda}_{d1} - \tilde{\gamma}_1}{(1 - \tilde{\lambda}_{d1})t_1 + \tilde{\lambda}_{d1} - \tilde{\gamma}_1 + \frac{\alpha_1}{\alpha_2}(1 - \tilde{\gamma}_1)}, \tag{96}$$

$$\frac{L_1}{L_1+L_2}=\frac{\frac{1}{\alpha_2}\left(\alpha_1-\tilde{\gamma}_1\right)+\tilde{\gamma}_1}{\left(1-\tilde{\lambda}_{d1}\right)t_1+\tilde{\lambda}_{d1}+\frac{1}{\alpha_2}\left(\alpha_1-\tilde{\gamma}_1\right)}. \tag{97}$$

A.4 Income and Intermediate Demand

The tariff formula (84) can also be used to derive an alternative expression for income I, which depends on tariff revenue given by $B=(t_1-1)\frac{\lambda_{m1}}{t_1}Y_1$. From trade balance we have $\frac{\lambda_{m1}Y_1}{t_1}=\lambda_{x1}Y_1^*$, and using (72) and (91) we obtain

$$\begin{aligned}
B=(t_1-1)\lambda_{x1}Y_1^* &= (t_1-1)\varphi_{x1}^{-\theta_1}N_1^e\left(\sigma_1 w f_{x1}\right)\left(\frac{\theta_1}{\theta_1+1-\sigma_1}\right)\\
&= \frac{(t_1-1)\sigma_1(\sigma_1-1)f_{x1}}{(1+(1-\gamma_1)(\sigma_1-1))(\theta_1+1-\sigma_1)f_1^e}wL_1\varphi_{x1}^{-\theta_1}.
\end{aligned}$$

Then income $I=wL+B$ equals

$$I=wL+(t_1-1)\frac{\sigma_1}{1+(1-\gamma_1)(\sigma_1-1)}\frac{f_{x1}{\varphi_{x1}}^{-\theta_1}}{\frac{(\theta_1+1-\sigma_1)}{(\sigma_1-1)}f_1^e}wL_1=wL+(t_1-1)\left(\frac{1-\tilde{\lambda}_{d1}}{1-\tilde{\gamma}_1}\right)wL_1.$$

Combining with (97), we have

$$\begin{aligned}
\frac{I}{wL_1} &= \frac{(1-\tilde{\lambda}_{d1})t_1+\tilde{\lambda}_{d1}+\frac{1}{\alpha_2}\left(\alpha_1-\tilde{\gamma}_1\right)}{\frac{1}{\alpha_2}\left(\alpha_1-\tilde{\gamma}_1\right)+\tilde{\gamma}_1}+(t_1-1)\left(\frac{1}{1-\tilde{\gamma}_1}\right)(1-\tilde{\lambda}_{d1}),\\
&= \frac{\alpha_2(t_1-1)\left(1-\tilde{\lambda}_{d1}\right)+1-\tilde{\gamma}_1}{\alpha_1(1-\tilde{\gamma}_1)}+(t_1-1)\frac{\left(1-\tilde{\lambda}_{d1}\right)}{1-\tilde{\gamma}_1}\\
&= \frac{1}{\alpha_1}+\frac{(t_1-1)\left(1-\tilde{\lambda}_{d1}\right)}{\alpha_1(1-\tilde{\gamma}_1)}.
\end{aligned}$$

Using $T(t_1)\equiv 1-\tilde{\gamma}_1+(t_1-1)(1-\tilde{\lambda}_{d1})$ from (86), we then obtain

$$I=\frac{wL_1}{\alpha_1}\frac{T(t_1)}{(1-\tilde{\gamma}_1)}. \tag{98}$$

Next, we derive the expression for the value of the finished goods used as an intermediate input in sector 1. We can rewrite the market clearing (6) as $Y_1=\alpha_1(wL+B)+\tilde{\gamma}_1(E_{d1}+E_{x1})$, where the second term is the demand for the finished good from firms. This intermediate demand is calculated as

$$\tilde{\gamma}_1(E_{d1}+E_{x1}) = N_1^e \gamma_1 \left(\int\limits_{\varphi_{d1}}^{\infty} \frac{c_1 q_{d1}(\varphi)}{\varphi} g_1(\varphi)\, d\varphi + \int\limits_{\varphi_{x1}}^{\infty} \frac{c_1 \tau_{x1} q_{x1}(\varphi)}{\varphi} g_1(\varphi)\, d\varphi \right). \tag{99}$$

Using the expression for profits, $\frac{c_1 \tau_{x1} q_{x1}(\varphi)}{\varphi} = (\sigma_1 - 1)\, \pi_{x1}(\varphi) + (\sigma_1 - 1)\, w f_{x1}$, we obtain:

$$\tilde{\gamma}_1(E_{d1}+E_{x1}) = N_1^e \gamma_1 (\sigma_1 - 1) \sum_{k=d,x} \int\limits_{\varphi_{k1}}^{\infty} (\pi_{k1}(\varphi) + w f_{k1})\, g_1(\varphi)\, d\varphi. \tag{100}$$

Using the free entry condition (76), we have

$$\tilde{\gamma}_1(E_{d1}+E_{x1}) = N_1^e \gamma_1 (\sigma_1 - 1)\, w \left(f_1^e + f_{d1} \varphi_{d1}^{-\theta_1} + f_{x1} \varphi_{x1}^{-\theta_1} \right).$$

Using labor market clearing (89), the intermediate demand is then given by

$$\tilde{\gamma}_1(E_{d1}+E_{x1}) = w L_1 \frac{\gamma_1 (\sigma_1 - 1)}{(1 + (1-\gamma_1)(\sigma_1 - 1))}.$$

It follows that the total demand for finished goods in sector 1 is

$$Y_1 = \alpha_1 I + w L_1 \frac{\gamma_1 (\sigma_1 - 1)}{(1 + (1-\gamma_1)(\sigma_1 - 1))}.$$

After combining these expressions with (70) and (75), I is given in terms of sector 1 variables by

$$I = \frac{w L_1}{\alpha_1} \frac{(\sigma_1 - 1)}{(1 + (1-\gamma_1)(\sigma_1 - 1))} \left(K_1^{\sigma_1 - 1} \frac{\sigma_1}{\theta_1 f_1^e} \left(\varphi_{d1}^{-\theta_1} f_{d1} + \varphi_{x1}^{-\theta_1} f_{x1} t_1 \right) - \gamma_1 \right). \tag{101}$$

For sector 2, there are no exports so that dividing the numerator and denominator of the ratio $\frac{(\sigma_2 - 1)}{1 + (1-\gamma_2)(\sigma_2 - 1)}$ by σ_2 to obtain $\frac{\rho_2}{1 - \tilde{\gamma}_2}$, then income can also be written in terms of sector 2 variables as

$$I = \frac{w L_2}{\alpha_2} \frac{\rho_2}{1 - \tilde{\gamma}_2} \left(K_2^{\sigma_2 - 1} \frac{\sigma_2}{\theta_2 f_2^e} \varphi_{d2}^{* - \theta_2} f_{d2} - (\gamma_2) \right) = \frac{w L_2}{\alpha_2}, \tag{102}$$

because $K_2^{\sigma_2 - 1} \left(\frac{\sigma_2 - 1}{\theta_2 f_2^e} \right) \varphi_{d2}^{* - \theta_2} f_{d2} = 1$ from (67) and (80).

A.5 Equilibrium Conditions

We use the definition of the SOE following Demidova and Rodríguez-Clare (2013). In particular the wages, prices, entry, and expenditure of the foreign country are not affected by changes in the home tariff. Formally, the equilibrium conditions of the SOE are as follows.

Definition 1. *An equilibrium of small open economy two-sector roundabout model, using domestic labor for fixed costs, is characterized for a set of prices* (w, c_1, c_2, P_1, P_2)*, productivity cutoffs* $(\varphi_{d1}, \varphi_{d2},\ \varphi_{x1}, \varphi^*_{x1})$*, finished outputs* (Y_1, Y_2)*, mass of firms* $\left(N^e_{1,} N^e_2\right)$*, and expenditure shares* $(\lambda_{d1}, \lambda_{m1}, \lambda_{x1})$ *that solve the following conditions taking as given* $\{P^*_1, Y^*_1, N^{e*}_1, c^*_1, w^* \equiv 1\}$*:*

Zero cut-off productivity (ZCP) *from (64)–(66),*

$$\varphi_{ds} = \left(\frac{\sigma_s}{\sigma_s - 1}\right)\left(\frac{\sigma_s w f_{ds}}{Y_s}\right)^{\frac{1}{\sigma_s - 1}} \frac{c_s}{P_s},\ \ s = 1, 2,$$

$$\varphi_{x1} = \left(\frac{\sigma_1}{\sigma_1 - 1}\right)\left(\frac{\sigma_1 w f_{x1}}{Y^*_1}\right)^{\frac{1}{\sigma_1 - 1}} \frac{c_1 \tau_{x1}}{P^*_1},$$

$$\varphi^*_{x1} = \left(\frac{\sigma_1}{\sigma_1 - 1}\right)\left(\frac{\sigma_1 w^* f^*_{x1}}{Y_1}\right)^{\frac{1}{\sigma_1 - 1}} \frac{c^*_1 \tau^*_{x1} (t_1)^{\frac{\sigma_1}{\sigma_1 - 1}}}{P_1},$$

Input cost indexes *from (1),*

$$c_s = w^{(1-\gamma_s)} P_s^{\gamma_s},\ \ s = 1, 2,$$

Value of finished output *from (8) and (94),*

$$Y_1 = \frac{\alpha_1 I}{1 - \tilde{\gamma}_1 \Lambda_1},$$

$$Y_2 = \frac{\alpha_2}{1 - \tilde{\gamma}_2} I,$$

with $I = wL + B = wL + (1 - \Lambda_1) Y_1,\ \Lambda_1 \equiv \left(\lambda_{d1} + \frac{\lambda_{m1}}{t_1}\right)$ *and* $\tilde{\gamma}_s = \left(\frac{\sigma_s - 1}{\sigma_s}\right) \gamma_s,\ s = 1, 2,$

Price indexes *from (69),*

$$P_1 = \left(\varphi_{d1}^{-\theta_1} N^e_1 \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c_1}{\bar{\varphi}_{d1}}\right)^{1-\sigma_1} + \varphi^{*\,-\theta_1}_{x1} N^{e*}_1 \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c^*_1\, \tau^*_{x1}\, t_1}{\bar{\varphi}^*_{x1}}\right)^{1-\sigma_1}\right)^{\frac{1}{1-\sigma_1}},$$

$$P_2 = \left(\varphi_{d2}^{-\theta_2} N_2^e \left(\frac{\sigma_2}{\sigma_2 - 1} \frac{c_2}{\bar{\varphi}_{d2}} \right)^{1-\sigma_2} \right)^{\frac{1}{1-\sigma_2}},$$

Entry *from (78) and (79),*

$$N_1^e = \frac{\Lambda_1 Y_1}{w f_1^e \left(\frac{\theta_1 \sigma_1}{\sigma_1 - 1} \right)},$$

$$N_2^e = \frac{Y_2}{w f_2^e \left(\frac{\theta_2 \sigma_2}{\sigma_2 - 1} \right)},$$

Expenditure share *from (71) and (72),*

$$\lambda_{d1} = \varphi_{d1}^{-\theta_1} N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c_1}{\bar{\varphi}_{d1} P_1} \right)^{1-\sigma_1} \quad \textit{and} \quad \lambda_{m1} = 1 - \lambda_{d1},$$

$$\lambda_{x1} = \varphi_{x1}^{-\theta_1} N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c_1 \tau_{x1}}{\bar{\varphi}_{x1} P_1^*} \right)^{1-\sigma_1},$$

with $\bar{\varphi}_{ks} = K_s \varphi_{ks}$, $K_s \equiv \left(\frac{\theta_s}{\theta_s + 1 - \sigma_s} \right)^{\frac{1}{\sigma_s - 1}}$, *for* $k = d, x$, *and* $\lambda_{d2} \equiv 1$,

Trade balance *from (74),*

$$\lambda_{x1} Y_1^* = \frac{\lambda_{m1} Y_1}{t_1}.$$

Using the second equalities in (71) and (73), we can substitute those expressions into the trade balance condition, to obtain the alternative condition used throughout the Appendix:

$$\varphi_{x1}^{-\theta_1} N_1^e w f_{x1} = \varphi_{x1}^{*\,-\theta_1} N_1^{e*} w^* f_{x1}^*.$$

A.6 Expenditure Shares and Trade Balance

We want to justify the share formulas that appear in (71)– (73). For the domestic share, the formula in Definition 1 appears as the first equality in (71). We use the average productivity which equals $\bar{\varphi}_{d1} = \varphi_{d1} \left(\frac{\theta_1}{\theta_1 + 1 - \sigma_1} \right)^{\frac{1}{\sigma_1 - 1}}$, from (67). Substituting this into (71), we obtain

$$\lambda_{d1} = \varphi_{d1}^{-\theta_1} N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c_1}{\varphi_{d1} P_1} \right)^{1-\sigma_1} \left(\frac{\theta_1}{\theta_1 + 1 - \sigma_1} \right).$$

We note that the first expression above in parentheses is the revenue earned by firms per variety and per dollar of expenditure Y_1. That magnitude equals σ_1 times fixed operating costs, so we can express the domestic share in the alternative form

$$\lambda_{d1} = \varphi_{d1}^{-\theta_1} N_1^e \left(\frac{\sigma_1 w f_{d1}}{Y_1}\right)\left(\frac{\theta_1}{\theta_1 + 1 - \sigma_1}\right),$$

which explains how we obtain the second equality in (71). The cutoff productivity for domestic firms is shown in (64), and substituting this above we obtain

$$\lambda_{d1} = N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1}\frac{c_1}{P_1}\right)^{-\theta_1} \left(\frac{\sigma_1 w f_{d1}}{Y_1}\right)^{1-\frac{\theta_1}{(\sigma_1-1)}} \left(\frac{\theta_1}{\theta_1 + 1 - \sigma_1}\right). \tag{103}$$

Notice that this expression is the same as (22) in the homogeneous firm model provided that we use the parameter restriction (11), except that there is a constant at the end preceded by a term involving the domestic firm's fixed costs relative to the value of output, wf_d/Y_1. That term captures the selection effect of an increase in real output Y_1/w lowering the cutoff productivity and raising the domestic share.[35] Inverting the above expression, and using the input cost index in (1), we obtain

$$P_1 = w\left(\frac{\lambda_{d1}}{N_1^e}\right)^{\frac{1}{\theta_1(1-\gamma_1)}} \left(\frac{\sigma_1 w f_{d1}}{Y_1}\right)^{\frac{\theta_1-\sigma_1+1}{\theta_1(1-\gamma_1)(\sigma_1-1)}} \left(\frac{\sigma_1}{\sigma_1 - 1}\right)^{\frac{1}{(1-\gamma_1)}} \left(\frac{\theta_1}{\theta_1 - \sigma_1 + 1}\right)^{\frac{-1}{\theta_1(1-\gamma_1)}}. \tag{104}$$

Replacing λ_{d1} with $1 - \lambda_{m1}$ and substituting back in the input cost index in (1), then totally differentiating, we obtain the change in marginal costs in (26) of the main text.

Now consider the share of home exporters in the foreign market. We start with the formula in Definition 1 which appears as the first equality in (72), and then use the average productivity

[35] The middle expression of (103), i.e., wf_{d1}/Y_1, also appears in Costinot and Rodríguez-Clare (2014). They adopt a wide range of specifications for fixed cost wf_{d1}, one of which makes it proportional to output Y_1 so this middle term is constant. To justify that specification of fixed costs, note that when comparing autarky and free trade there is no tariff revenue, so we could specify wf_{d1} as a fraction of home income I, or total factor income. In the absence of roundabout production, the market clearing condition (4) shows that $Y_1 = \alpha_1 I$, so in that case we conclude that the numerator and denominator of wf_{d1}/Y_1 are both proportional to I, so this ratio is constant.

which equals $\bar{\varphi}_{x1} = \varphi_{x1}\left(\frac{\theta_1}{\theta_1+1-\sigma_1}\right)^{\frac{1}{\sigma_1-1}}$ from (67). Substituting this into (72), we obtain

$$\lambda_{x1} = \varphi_{x1}^{-\theta_1} N_1^e \left(\frac{\sigma_1}{\sigma_1-1}\frac{c_1\tau_{x1}}{\varphi_{x1}P_1^*}\right)^{1-\sigma_1}\left(\frac{\theta_1}{\theta_1+1-\sigma_1}\right).$$

We note that the first expression above in parentheses is the revenue earned by exporters per variety and per dollar of foreign expenditure Y_1^*. That magnitude equals σ_1 times fixed operating costs, so we can express the export share in the alternative form

$$\lambda_{x1} = \varphi_{x1}^{-\theta_1}\, N_1^e \left(\frac{\sigma_1 w f_{x1}}{Y_1^*}\right)\left(\frac{\theta_1}{\theta_1+1-\sigma_1}\right),$$

which explains how we obtain the second equality in (72). The cutoff productivity for home exporters is shown in (65), and substituting this above we obtain (28) in the main text.

Third, consider the import share of foreign exporters in the home market. That import share is obtained in Definition 1 from $\lambda_{m1} = 1 - \lambda_{d1}$, and an explicit equation for this share is obtained by using the price index P_1 in λ_{d1}, and simplifying to obtain

$$\lambda_{m1} = \varphi_{x1}^{*\,-\theta_1}\, N_1^{*e}\left(\frac{\sigma_1}{\sigma_1-1}\,\frac{c_1^*\,\tau_{x1}\,t_1}{\bar{\varphi}_{x1}^*\,P_1}\right)^{1-\sigma_1}, \tag{105}$$

as was shown in the first equality of (73). We use the average productivity which equals $\bar{\varphi}_{x1}^* = \varphi_{x1}^*\left(\frac{\theta_1}{\theta_1+1-\sigma_1}\right)^{\frac{1}{\sigma_1-1}}$. Substituting this into (105), we obtain

$$\lambda_{m1} = \varphi_{x1}^{*\,-\theta_1} N_1^e \left(\frac{\sigma_1}{\sigma_1-1}\frac{c_1^*\,\tau_x^*\,t_1}{\varphi_{x1}^* P_1}\right)^{1-\sigma_1}\left(\frac{\theta_1}{\theta_1+1-\sigma_1}\right).$$

The first expression above in parentheses is the revenue earned by foreign exporters per variety and per dollar of home expenditure Y_1. That magnitude equals σ_1 times fixed operating costs, so we can express the import share in the alternative form

$$\lambda_{m1} = \varphi_{x1}^{-\theta_1}\, N_1^e \left(\frac{\sigma_1 w f_{x1}}{Y_1^*}\right)\left(\frac{\theta_1}{\theta_1+1-\sigma_1}\right), \tag{106}$$

which explains how we obtain the second equality in (73). The cutoff productivity for foreign exporters is shown in (66), and substituting this into (73) we obtain

$$\lambda_{m1} = N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c_1^* \tau_x^* t_1}{P_1} \right)^{-\theta_1} \left(\frac{\sigma_1 w^* f_{x1}^* t_1}{Y_1} \right)^{1 - \frac{\theta_1}{(\sigma_1 - 1)}} \left(\frac{\theta_1}{\theta_1 + 1 - \sigma_1} \right). \tag{107}$$

The middle term on the right of (107) is the impact of selection on changing the cutoff productivity of foreign exporters and therefore changing their import share at home. Consider the case with little roundabout, i.e., $\alpha_1 > \tilde{\gamma}_1$. Then from our discussion in section 2.1 we know that Y_1/w rises with a small tariff starting from free trade, so with the tariff increasing then the wage and Y_1 both rise. It follows from (107) that λ_{m1} rises as the cutoff for foreign firms falls. In conjunction with the effect of selection on raising the export share, as discussed in the main text, this will tend to restore equilibrium in the balance of trade. It follows that the needed increase in the wage to restore equilibrium is reduced, which intuitively explains result (31) in the main text.[36]

B Two-Sector Small Open Homogeneous Economy Model

The structure of the economy is still as illustrated in Figure 1, and the description in the main text section 2 continues to apply. We drop the index φ from firms since the productivities are all set at unity, and then equation (53) is revised to become $Q_{d1} \equiv (N_1^e)^{\frac{\sigma_1 - 1}{\sigma_1}} q_{d1}$ and $Q_{m1} \equiv (N_1^{e*})^{\frac{\sigma_1 - 1}{\sigma_1}} q_{x1}^*$ while (54) is revised to become $P_{d1} \equiv (N_1^e)^{\frac{1}{1-\sigma_1}} p_{d1}$ and $P_{x1}^* \equiv (N_1^{e*})^{\frac{1}{1-\sigma_1}} p_{x1}^*$. It follows that the home price index is:

$$P_1 = \left(N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1} c_1 \right)^{1-\sigma_1} + N_1^{*e} \left(\frac{\sigma_1}{\sigma_1 - 1} c_1^* \tau_{x1}^* t_1 \right)^{1-\sigma_1} \right)^{\frac{1}{1-\sigma_1}}, \tag{108}$$

and similarly in sector 2 but without the imported varieties.

The expressions for quantities, profits and prices in (56)–(63) continue to hold, except that all productivities are set at unity. In addition, the *first equality* in each of the share equations

[36] Notice that the tariff t_1 also multiplies the value of foreign fixed costs in the middle term of (107), which occurs because we have modeled the tariff as applying to the c.i.f. value of imports, inclusive of costs, freight and markups (see note 34). That tariff in the middle term leads to greater selection of foreign exporters and reduces the home import share, which works in the opposite direction of the increase in Y_1 raising the import share. Nevertheless, when all effects are taken into account, we obtain a reduced terms of trade impact of the tariff due to heterogeneous firms in the absence of roundabout production, as shown in (31).

(71)–(73) hold with all productivities set at unity. Other equations for equilibrium entry, and the labor allocations across sectors can be derived analogous to those in Appendix A, and we record all the equilibrium conditions with homogeneous firms in the following definition.

Definition 2. *An equilibrium of small open economy, two-sector roundabout model with homogeneous firms, using domestic labor for fixed costs, is characterized for a set of prices* (w, c_1, c_2, P_1, P_2)*, finished outputs* (Y_1, Y_2)*, mass of firms* (N_1^e, N_2^e)*, and expenditure shares* $(\lambda_{d1}, \lambda_{m1}, \lambda_{x1})$ *that solve the following equilibrium conditions taking as given* $\{P_1^*, Y_1^*, N_1^{e*}, c_1^*, w^* \equiv 1\}$*:*

Input cost indexes *from (1),*

$$c_s = w^{(1-\gamma_s)} P_s^{\gamma_s}, \ s = 1, 2,$$

Value of finished output *from (8) and (94),*

$$Y_1 = \frac{\alpha_1 I}{1 - \tilde{\gamma}_1 \Lambda_1},$$

$$Y_2 = \frac{\alpha_2}{1 - \tilde{\gamma}_2} I,$$

with $I = wL + B = wL + (1 - \Lambda_1) Y_1$*,* $\Lambda_1 \equiv \left(\lambda_{d1} + \frac{\lambda_{m1}}{t_1}\right)$ *and* $\tilde{\gamma}_s = \left(\frac{\sigma_s - 1}{\sigma_s}\right) \gamma_s$*,* $s = 1, 2,$

Price indexes *from (108),*

$$P_1 = \left(N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1} c_1 \right)^{1-\sigma_1} + N_1^{e*} \left(\frac{\sigma_1}{\sigma_1 - 1} c_1^* \tau_{x1}^* t_1 \right)^{1-\sigma_1} \right)^{\frac{1}{1-\sigma_1}},$$

$$P_2 = \left(N_2^e \left(\frac{\sigma_2}{\sigma_2 - 1} c_2 \right)^{1-\sigma_2} \right)^{\frac{1}{1-\sigma_2}},$$

Entry *from (78) and (79),*

$$N_1^e = \frac{\Lambda_1 Y_1}{w f_1^e \sigma_1} = \frac{\Lambda_1}{w f_1^e \sigma_1} \frac{\alpha_1 I}{1 - \tilde{\gamma}_1 \Lambda_1},$$

$$N_2^e = \frac{Y_2}{w f_2^e \sigma_2} = \frac{1}{w f_2^e \sigma_2} \frac{\alpha_2}{1 - \tilde{\gamma}_2} I,$$

Expenditure share *from (71), (72) and (73)*

$$\lambda_{d1} = N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c_1}{P_1} \right)^{1-\sigma_1} \quad \text{and} \quad \lambda_{m1} = 1 - \lambda_{d1},$$

$$\lambda_{x1} = N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c_1}{P_1^*} \right)^{1-\sigma_1},$$

Trade balance *from (74),*

$$\lambda_{x1} Y_1^* = \frac{\lambda_{m1} Y_1}{t_1}.$$

In section 2.1, we state that the impact of the tariff on entry with homogeneous firms is the same as with heterogeneous firms. To justify this claim, use Definition 2 to obtain sector 1 output

$$Y_1 = \frac{\alpha_1 (wL + (1 - \Lambda_1) Y_1)}{1 - \tilde{\gamma}_1 \Lambda_1} \implies Y_1 = \frac{\alpha_1 wL}{[\alpha_2 + (\alpha_1 - \tilde{\gamma}_1) \Lambda_1]}.$$

It follows that entry into sector 1 is given by

$$N_1^e = \frac{\Lambda_1 Y_1}{w f_1^e \sigma_1} = \frac{\Lambda_1 \alpha_1 L}{w f_1^e \sigma_1 [\alpha_2 + (\alpha_1 - \tilde{\gamma}_1) \Lambda_1]},$$

which is the same as shown in (10) for the heterogeneous firm model when we adopt the parameter restriction in (11). Likewise for entry into sector 2.

C Closed Economy Model

In the closed-economy model we allow for multiple sectors $s = 1, ..., S$, where we use $\alpha_s > 0$ to denote the consumption share in each sector with with $\sum_{s=1}^{S} \alpha_s = 1$. We now introduce producer and consumer tax/subsidies t_s^p and t_s^c on purchases of the finished good. The producer tax/subsidy means that the input cost index is modified from (1) as

$$c_s = w^{(1-\gamma_s)} (t_s^p P_s)^{\gamma_s}, \tag{109}$$

where P_s denotes the price of the finished good before the application of any tax/subsidies.

Without loss of generality, we assume that the government budget is balanced so that $B = 0$. In the market clearing condition (4), there is no trade so that $\lambda_{ds} = 1$ and $\lambda_{xs} = 0$, and the consumer and firm purchases must be divided by t_s^c and t_s^p, respectively, to obtain the net-of-tax purchases. Further multiplying these purchases by the *ad valorem* tax rates $t_s^c - 1$ and $t_s^p - 1$, respectively, we obtain the balanced budget

$$0 = \sum_{s=1}^{S} (t_s^c - 1) \frac{\alpha_s w L}{t_s^c} + (t_s^p - 1) \frac{\tilde{\gamma}_s Y_s}{t_s^p}. \tag{110}$$

The term $\alpha_s wL/t_s^c$ on the right of (110) is the value of consumer purchases of the finished good. Dividing this by the duty-free price index of the finished good, P_s, we obtain consumption in each sector, and so the objective function for the government is

$$\max_{t_s^c, t_s^p > 0} \prod_{s=1}^{S} C_s^{\alpha_s} = \prod_{s=1}^{S} \left(\frac{\alpha_s w L}{t_s^c P_s} \right)^{\alpha_s}, \tag{111}$$

subject to the constraint (110).

To determine the optimal policies, we need an expression for the price index in each sector under autarky. Recall from (54) that P_{ds} is the CES price index for differentiated inputs purchased from domestic firms in each sector. Using the input price index (109), we can substitute prices from (62) into (54) to obtain

$$P_{ds} = (N_{ds})^{\frac{1}{(1-\sigma_s)}} \left(\frac{\sigma_s}{\sigma_s - 1} \right) \frac{w^{(1-\gamma_s)} \left(t_s^p P_s\right)^{\gamma_s}}{\bar{\varphi}_s}. \tag{112}$$

In a closed economy we have $P_{ds} = P_s$, and so we can solve for the price index P_s from (112)as

$$P_s = w \left[\left(\frac{1}{N_{ds}} \right)^{\frac{1}{(\sigma_s - 1)}} \left(\frac{\sigma_s}{\sigma_s - 1} \right) \frac{(t_s^p)^{\gamma_s}}{\bar{\varphi}_{ds}} \right]^{\frac{1}{(1-\gamma_s)}}. \tag{113}$$

This expression includes the average productivities, but these are not affected by the consumer or producer taxes because from (67) they are proportional to the cutoff productivities, which are determined by the free-entry condition like (80) but in each sector: $J_s(\varphi_{ds}) f_{ds} = f_s^e$. It follows that the cutoffs are not affected by the tax/subsidy instruments.

Entry into all sectors is endogenous. Using sector 1 as an example, N_1^e is determined by a modified entry condition (78), where the expenditure on the differentiated inputs in the closed economy, E_{d1}, equals the net-of-tax value of the final good that are bundled from them, Y_1, and we ignore the term E_{x1}. In the market clearing condition (4), with no trade then $\lambda_{d1} = 1$ and

$\lambda_{x1} = 0$, and the consumer and firm purchases must be divided by t_s^c and t_s^p, respectively, to obtain the net-of-tax purchases in all sectors

$$Y_s = \frac{\alpha_s}{t_s^c} wL + \frac{\tilde{\gamma}_s}{t_s^p} Y_s,$$

recalling that we have set $B = 0$ so that wL is consumer income. We solve for $Y_s = \frac{\alpha_s wL}{t_s^c[1-(\tilde{\gamma}_s/t_s^p)]}$, and then entry from (78) is

$$N_s^e = (\alpha_s L) \Bigg/ \left[t_s^c \left(1 - \frac{\tilde{\gamma}_s}{t_s^p} \right) f_s^e \left(\frac{\theta_s \sigma_s}{\sigma_s - 1} \right) \right]. \tag{114}$$

Substituting (114) into (55), (113) and then (111) and ignoring constants, the objective function is

$$\max_{t_s^c, t_s^p > 0} \prod_{s=1}^{S} \left\{ t_s^c \left[t_s^c \left(1 - \frac{\tilde{\gamma}_s}{t_s^p} \right) \right]^{\frac{1}{(1-\gamma_s)(\sigma_s - 1)}} (t_s^p)^{\frac{\gamma_s}{(1-\gamma_s)}} \right\}^{-\alpha_s}. \tag{115}$$

We solve the problem (115) subject to (110) twice: in the first-best by choosing the optimal consumer and producer tax/subsidies; and in the second-best by choosing t_s^c while setting $t_s^p \equiv 1$. The solutions are shown in (12) and (16), respectively, for the case of just two sectors.

D Fixed-point Formula for the Second-Best Tariff

We now assume that no consumer or producer tax/subsidies apply to purchases of the finished good in either sector, and the only policy instrument used is the tariff t_1 on imports of the differentiated inputs in sector 1. In this Appendix we perform the comparative statics with respect to a change in the tariff t_1 to obtain the fixed-point formula for the optimal tariff (37), and in Appendix E we develop the proof of Theorem 1.

We first derive an expression for the price index that is going to be used in order to express welfare as a function of productivity thresholds. From (69) the sector 1 price index is

$$P_1 = \left(\varphi_{d1}^{-\theta_1} N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c_1}{\bar{\varphi}_{d1}} \right)^{1-\sigma_1} + \varphi_{x1}^{*\,-\theta_1} N_1^{e*} \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{c_1^* \tau_{x1}^* t_1}{\bar{\varphi}_{x1}^*} \right)^{1-\sigma_1} \right)^{\frac{1}{1-\sigma_1}}.$$

We use the entry thresholds φ_{x1}^{*} and φ_{d1} in Definition 1 to obtain

$$\left(\frac{\sigma_1}{\sigma_1-1}\frac{c_1^{*}\,\tau_{x1}^{*}\,t_1}{\varphi_{x1}^{*}}\right)^{1-\sigma_1}=\frac{\sigma_1\,w^{*}\,f_{x1}^{*}\;t_1}{\sigma_1\,w\,f_{d1}}\left(\frac{\sigma_1}{\sigma_1-1}\frac{c_1}{\varphi_{d1}}\right)^{1-\sigma_1}. \tag{116}$$

Using this expression together with trade balance (75), with (67) and (1) , we obtain

$$P_1=\left(\frac{\sigma_1}{\sigma_1-1}\frac{1}{K_1}\left(\frac{N_1^e}{f_{d1}}\right)^{\frac{1}{(1-\sigma_1)}}\right)^{\frac{1}{(1-\gamma_1)}}\varphi_{d1}^{-\frac{1}{(1-\gamma_1)}}w\left(\varphi_{d1}^{-\theta_1}\,f_{d1}+\,\varphi_{x1}^{-\theta_1}\;f_{x1}t_1\right)^{\frac{1}{(1-\gamma_1)(1-\sigma_1)}}. \tag{117}$$

Similarly, we obtain

$$P_2=\left(\frac{\sigma_2}{\sigma_2-1}\frac{1}{K_2}\left(\frac{N_2^e}{f_{d2}}\right)^{\frac{1}{(1-\sigma_2)}}\right)^{\frac{1}{(1-\gamma_2)}}\varphi_{d2}^{-\frac{1}{(1-\gamma_2)}}w\left(\varphi_{d2}^{-\theta_2}\,f_{d2}\right)^{\frac{1}{(1-\gamma_2)(1-\sigma_2)}}.$$

Using expressions (101) and (102) for income and the above expressions for the price indexes, we substitute these into indirect utility or welfare, which from the Cobb-Douglas utility function is given by

$$U=\left(\frac{\alpha_1 I}{P_1}\right)^{\alpha_1}\left(\frac{\alpha_2 I}{P_2}\right)^{\alpha_2}. \tag{118}$$

Define the term,

$$\Theta\equiv\left(\frac{\frac{(\sigma_1-1)}{(1+(1-\gamma_1)(\sigma_1-1))}}{\left(\frac{\sigma_1}{\sigma_1-1}\frac{1}{K_1}\left(\frac{1}{f_{d1}}\right)^{\frac{1}{(1-\sigma_1)}}\right)^{\frac{1-\sigma_1}{1-\gamma_1(1-\sigma_1)}}}\right)^{\alpha_1}\left(\frac{1}{\left(\frac{\sigma_2}{\sigma_2-1}\frac{1}{K_2}\left(\frac{1}{f_{d2}}\right)^{\frac{1}{(1-\sigma_2)}}\right)^{\frac{1}{(1-\gamma_2)}}(\varphi_{d2})^{-\frac{1}{1-\gamma_2}}\left(f_{d2}\;\varphi_{d2}^{-\theta_2}\right)^{\frac{1}{(1-\gamma_2)(1-\sigma_2)}}}\right)^{\alpha_2}$$

which is a constant because φ_{d2} is constant from (80). We then obtain the welfare expression,

$$\begin{aligned}U\;&=\;\Theta\left(\frac{\frac{K_1^{\sigma_1-1}\sigma_1}{\theta_1 f_1^e}\left(\varphi_{d1}^{-\theta_1}\;f_{d1}+\;\;\varphi_{x1}{}^{-\theta_1}\;f_{x1}\,t_1\right)-\gamma_1}{\left(f_{d1}\varphi_{d1}^{-\theta_1}+\varphi_{x1}^{-\theta_1}\;f_{x1}t_1\right)^{\frac{1}{(1-\gamma_1)(1-\sigma_1)}}}(\varphi_{d1})^{\frac{1}{(1-\gamma_1)}}\right)^{\alpha_1}\left(\frac{L_1}{(N_1^e)^{\frac{1}{(1-\gamma_1)(1-\sigma_1)}}}\right)^{\alpha_1}\\&\quad\times\left(\frac{L_2}{(N_2^e)^{\frac{1}{(1-\gamma_2)(1-\sigma_2)}}}\right)^{\alpha_2}.\end{aligned}$$

There is new term in this expression, given by

$$\left(\frac{L_1}{(N_1^e)^{\frac{1}{(1-\gamma_1)(1-\sigma_1)}}}\right)^{\alpha_1}\left(\frac{L_2}{(N_2^e)^{\frac{1}{(1-\gamma_2)(1-\sigma_2)}}}\right)^{\alpha_2} =$$
$$\left(\frac{(1+(1-\gamma_1)(\sigma_1-1))\,\theta_1 f_1^e}{(\sigma_1-1)}\,(N_1^e)^{\frac{1+(1-\gamma_1)(\sigma_1-1)}{(1-\gamma_1)(\sigma_1-1)}}\right)^{\alpha_1}\left(\frac{(1+((1-\gamma_2))(\sigma_2-1))\,\theta_2 f_1^e}{(\sigma_2-1)}\,(N_2^e)^{\frac{1+(1-\gamma_2)(\sigma_2-1)}{(1-\gamma_2)(\sigma_2-1)}}\right)^{\alpha_2}$$

using (91) and (92).

Totally differentiating welfare, $\hat{U}$ can be written as

$$\begin{aligned}
\hat{U} &= \alpha_1\left(1+\frac{\gamma_1}{\frac{\alpha_1 I}{wL_1}\frac{(1+(1-\gamma_1)(\sigma_1-1))}{(\sigma_1-1)}}+\frac{1}{(1-\gamma_1)(\sigma_1-1)}\right)\left((1-\lambda_{d1})\left(-\theta_1\hat{\varphi}_{x1}\hat{t}_1\right)-\theta_1\lambda_{d1}\hat{\varphi}_{d1}\right)\\
&\quad+\frac{\alpha_1}{(1-\gamma_1)}\hat{\varphi}_{d1}+\alpha_1\frac{1+(1-\gamma_1)(\sigma_1-1)}{(1-\gamma_1)(\sigma_1-1)}\hat{N}_1^e+\alpha_2\frac{1+1-\gamma_2(\sigma_2-1)}{(1-\gamma_2)(\sigma_2-1)}\hat{N}_2\\
&= \alpha_1\left(1+\frac{\gamma_1(\sigma_1-1)}{(1+(1-\gamma_1)(\sigma_1-1))+\sigma_1(t_1-1)(1-\tilde{\lambda}_{d1})}+\frac{1}{(1-\gamma_1)(\sigma_1-1)}\right)\\
&\quad\times\left((1-\lambda_{d1})\left(-\theta_1\hat{\varphi}_{x1}+\hat{t}_1\right)-\theta_1\lambda_{d1}\hat{\varphi}_{d1}\right)+\frac{1}{(1-\gamma_1)}\hat{\varphi}_{d1}\\
&\quad+\alpha_1\left[\frac{1+(1-\gamma_1)(\sigma_1-1)}{(1-\gamma_1)(\sigma_1-1)}\right]\hat{N}_1^e+\alpha_2\left[\frac{1+(1-\gamma_2)(\sigma_2-1)}{(1-\gamma_2)(\sigma_2-1)}\right]\hat{N}_2^e, \qquad (119)
\end{aligned}$$

where the equality is obtained by using the following expression

$$\begin{aligned}
1 &+ \frac{\gamma_1}{\frac{\alpha_1 I}{wL_1}\frac{(1+(1-\gamma_1)(\sigma_1-1))}{(\sigma_1-1)}}+\frac{1}{(1-\gamma_1)(\sigma_1-1)} = 1+\frac{\gamma_1}{\left(1+\frac{\sigma_1(t_1-1)(1-\tilde{\lambda}_{d1})}{(1+(1-\gamma_1)(\sigma_1-1))}\right)\frac{(1+(1-\gamma_1)(\sigma_1-1))}{(\sigma_1-1)}}\\
&+ \frac{1}{(1-\gamma_1)(\sigma_1-1)} = 1+\frac{\gamma_1(\sigma_1-1)}{(1+(1-\gamma_1)(\sigma_1-1))+\sigma_1(t_1-1)(1-\tilde{\lambda}_{d1})}+\frac{1}{(1-\gamma_1)(\sigma_1-1)}.
\end{aligned}$$

Now the strategy is to obtain expressions for $\hat{\varphi}_{d1}$ and $\hat{\varphi}_{x1}$. First, totally differentiate the free entry condition (78) and use (82) to obtain

$$\hat{\varphi}_{d1} = -\left(\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\right)\hat{\varphi}_{x1}. \qquad (120)$$

Then we totally differentiate the price index (117) to obtain

$$\begin{aligned}\hat{P}_1 &= \frac{1}{(1-\gamma_1)(1-\sigma_1)}\hat{N}_1^e + \hat{w} + \frac{1}{(1-\gamma_1)}\left(\frac{\left(1-\tilde{\lambda}_{d1}\right)}{\tilde{\lambda}_{d1}}\left(\frac{\theta_1}{(1-\sigma_1)}\lambda_{d1} + 1\right) - \frac{\theta_1(1-\lambda_{d1})}{(1-\sigma_1)}\right)\hat{\varphi}_{x1} \\ &+\frac{1}{(1-\gamma_1)(1-\sigma_1)}(1-\lambda_{d1})\hat{t}_1. \end{aligned} \tag{121}$$

Next, totally differentiate the expression for φ_{x1} in (65) and recall that the foreign price index, value of output and input-cost index are fixed. It follows that $\hat{\varphi}_{x1}$ is given by

$$\hat{\varphi}_{x1} - \left(\frac{1}{\sigma_1 - 1} + (1-\gamma_1)\right)\hat{w} = \gamma_1 \hat{P}_1. \tag{122}$$

Now combine (121) and (122) to obtain

$$\begin{aligned}&\frac{\hat{\varphi}_{x1}}{\gamma_1} - \frac{1}{\gamma_1}\left(\frac{1}{\sigma_1-1} + (1-\gamma_1)\right)\hat{w} = \\ &\frac{1}{(1-\gamma_1)(1-\sigma_1)}\hat{N}_1^e + \hat{w} + \frac{1}{(1-\gamma_1)}\left(\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{\theta_1\lambda_{d1}}{1-\sigma_1} + 1\right) - \frac{\theta_1(1-\lambda_{d1})}{1-\sigma_1}\right)\hat{\varphi}_{x1} \\ &+\frac{1-\lambda_{d1}}{(1-\gamma_1)(1-\sigma_1)}\hat{t}_1. \end{aligned} \tag{123}$$

From trade balance (75), we have

$$\hat{\varphi}^*_{x1} = \hat{\varphi}_{x1} - \frac{1}{\theta_1}\hat{w} - \frac{1}{\theta_1}\hat{N}_1^e. \tag{124}$$

From the relationship between $\hat{\varphi}^*_{x1}$ and $\hat{\varphi}_{d1}$ in (116), we can see that

$$\hat{\varphi}^*_{x1} = \hat{\varphi}_{d1} - \left(\frac{1}{\sigma_1-1} + (1-\gamma_1)\right)\hat{w} - \gamma_1\hat{P}_1 + \frac{\sigma_1}{\sigma_1-1}\hat{t}_1.$$

Combining (120) and (122), then $\hat{\varphi}^*_{x1}$ is given by

$$\hat{\varphi}^*_{x1} = -\frac{1}{\tilde{\lambda}_{d1}}\left(\frac{1}{\sigma_1-1} + (1-\gamma_1)\right)\hat{w} - \frac{1}{\tilde{\lambda}_{d1}}\gamma_1\hat{P}_1 + \frac{\sigma_1}{\sigma_1-1}\hat{t}_1,$$

and after using (124), we obtain

$$\hat{\varphi}_{x1}-\frac{1}{\theta_1}\hat{w}-\frac{1}{\theta_1}\hat{N}_1^e=-\frac{1}{\tilde{\lambda}_{d1}}\left(\frac{1}{\sigma_1-1}+(1-\gamma_1)\right)\hat{w}-\frac{1}{\tilde{\lambda}_{d1}}\gamma_1\hat{P}_1+\frac{\sigma_1}{\sigma_1-1}\hat{t}_1.$$

Then from (122) we have

$$\hat{\varphi}_{x1}=\frac{\tilde{\lambda}_{d1}}{1+\tilde{\lambda}_{d1}}\frac{1}{\theta_1}\left(\hat{w}+\hat{N}_1^e\right)+\frac{\tilde{\lambda}_{d1}}{1+\tilde{\lambda}_{d1}}\frac{\sigma_1}{\sigma_1-1}\hat{t}_1. \tag{125}$$

Using (123) and multiplying both sides by γ_1 we have

$$\begin{aligned}-\left(\frac{\sigma_1}{\sigma_1-1}\right)\hat{w} \;&=\; \left(-1+\frac{\gamma_1}{(1-\gamma_1)}\left(\frac{\theta_1}{(1-\sigma_1)}\left(\frac{\lambda_{d1}-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\right)+\frac{(1-\tilde{\lambda}_{d1})}{\tilde{\lambda}_{d1}}\right)\right)\hat{\varphi}_{x1}\\ &+\; \frac{\gamma_1}{(1-\gamma_1)(1-\sigma_1)}\hat{N}_1^e+\frac{\gamma_1}{(1-\gamma_1)(1-\sigma_1)}(1-\lambda_{d1})\hat{t}_1. \end{aligned} \tag{126}$$

D.1 Impact of the Tariff on the Home Wage

D.1.1 Heterogeneous Firms

Combining expression (126) with (125) and using $\rho_1 \equiv \frac{\sigma_1-1}{\sigma_1}$, we finally obtain

$$\hat{w}=\mathcal{E}_1^{het}\hat{t}_1+\mathcal{E}_2^{het}\hat{N}_1^e,$$

where

$$\mathcal{E}_1^{het}=\frac{1-\frac{\gamma_1}{(1-\gamma_1)}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(1-\frac{1}{\sigma_1}\frac{1-\lambda_{d1}\tilde{\lambda}_{d1}}{1-\tilde{\lambda}_{d1}}+\left(\frac{1}{\sigma_1}-\frac{\theta_1}{\sigma_1-1}\right)(1-t_1)\lambda_{d1}\right)}{\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}-\frac{\rho_1}{\theta_1}+\frac{\gamma_1}{1-\gamma_1}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{\rho_1}{\theta_1}-\frac{1}{\sigma_1}(1-t_1)\lambda_{d1}\right)}, \tag{127}$$

$$\mathcal{E}_2^{het}=\frac{\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\sigma_1}\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}+\frac{\rho_1}{\theta_1}-\frac{\gamma_1}{(1-\gamma_1)}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{\rho_1}{\theta_1}-\frac{1}{\sigma_1}(1-t_1)\lambda_{d1}\right)}{\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}-\frac{\rho_1}{\theta_1}+\frac{\gamma_1}{(1-\gamma_1)}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{\rho_1}{\theta_1}-\frac{1}{\sigma_1}(1-t_1)\lambda_{d1}\right)}. \tag{128}$$

Notice that at free trade with $t_1=1$ and $\tilde{\lambda}_{d1}=\lambda_{d1}$, then we obtain the wage elasticity $\mathcal{E}_1^{het}(\gamma_1)$ where we now add the argument γ_1 and supercript the parameters ρ_1^{het} and σ_1^{het}:

$$\mathcal{E}_1^{het}(\gamma_1)=\frac{1-\frac{\gamma_1}{(1-\gamma_1)}\frac{1-\lambda_{d1}}{\lambda_{d1}}\left(1-\frac{1}{\sigma_1^{het}}\frac{1-\lambda_{d1}^2}{1-\lambda_{d1}}\right)}{\frac{1+\lambda_{d1}}{\lambda_{d1}}-\frac{\rho_1^{het}}{\theta_1}+\frac{\gamma_1}{(1-\gamma_1)}\frac{1-\lambda_{d1}}{\lambda_{d1}}\frac{\rho_1^{het}}{\theta_1}}. \tag{129}$$

The denominator of this expression is positive because $\theta_1 > \sigma_1^{het} - 1 > 0$, so it is immediate that $\mathcal{E}_1^{het}(0) > 0$. Simplifying slightly using $1 - \lambda_{d1}^2 = (1 - \lambda_{d1})(1 + \lambda_{d1})$ and using η_{m1} from (25), we see that $\mathcal{E}_1^{het}(\gamma_1) < 0$ if and only if

$$1 < \eta_{m1}\left(1 - \frac{1 + \lambda_{d1}}{\sigma_1^{het}}\right) \iff \eta_{m1} > \left(\frac{\sigma_1^{het}}{\sigma_1^{het} - 2 + \lambda_{m1}}\right). \tag{130}$$

Because η_{m1} is increasing in γ_1, we confirm that $\mathcal{E}_1^{het}(\gamma_1)$ is declining in γ_1 by checking its derivative with respect to η_{m1}:

$$\begin{aligned}
\frac{d\mathcal{E}_1^{het}}{d\eta_{m1}} &= -\frac{\left(\frac{\sigma_1^{het}-2+\lambda_{m1}}{\sigma_1^{het}}\right)\left[\frac{1+\lambda_{d1}}{\lambda_{d1}} + (\eta_{m1} - 1)\frac{\rho_1}{\theta_1}\right] + \frac{\rho_1}{\theta_1}\left[1 - \eta_{m1}\left(\frac{\sigma_1^{het}-2+\lambda_{m1}}{\sigma_1^{het}}\right)\right]}{\left[\frac{1+\lambda_{d1}}{\lambda_{d1}} + (\eta_{m1} - 1)\frac{\rho_1}{\theta_1}\right]^2} \\
&= -\frac{\left(\frac{\sigma_1^{het}-2+\lambda_{m1}}{\sigma_1^{het}}\right)\frac{1+\lambda_{d1}}{\lambda_{d1}} - \frac{\rho_1}{\theta_1}\left(\frac{\sigma_1^{het}-2+\lambda_{m1}}{\sigma_1^{het}}\right) + \frac{\rho_1}{\theta_1}}{\left[\frac{1+\lambda_{d1}}{\lambda_{d1}} + (\eta_{m1} - 1)\frac{\rho_1}{\theta_1}\right]^2} \\
&= -\frac{\left(\frac{\sigma_1^{het}-2+\lambda_{m1}}{\sigma_1^{het}}\right)\frac{1+\lambda_{d1}}{\lambda_{d1}} + \frac{\rho_1}{\theta_1}\left(\frac{2-\lambda_{m1}}{\sigma_1^{het}}\right)}{\left[\frac{1+\lambda_{d1}}{\lambda_{d1}} + (\eta_{m1} - 1)\frac{\rho_1}{\theta_1}\right]^2},
\end{aligned}$$

which is negative for $\sigma_1^{het} > 2$. This proves the statements about $\mathcal{E}_1^{het}(\gamma_1)$ in (30).

D.1.2 Homogeneous Firms

For comparison, we derive the change in wages with homogeneous firms. We begin by differentiating the price index (108) in sector 1 to obtain

$$\hat{P}_1 = \frac{1}{1 - \sigma_1}\lambda_{d1}\hat{N}_1^e + \lambda_{d1}\hat{c}_1 + (1 - \lambda_{d1})\,\hat{t}_1.$$

Using the input-cost index (1) so that $\hat{c}_1 = (1 - \gamma_1)\hat{w} + \gamma_1\hat{P}_1$, we can rewrite this expression as

$$\hat{P}_1 = \frac{1}{1 - \sigma_1}\frac{\lambda_{d1}}{(1 - \gamma_1\lambda_{d1})}\hat{N}_1 + \frac{1 - \gamma_1\lambda_{d1}}{1 - \gamma_1\lambda_{d1}}\hat{w} + \frac{(1 - \lambda_{d1})}{1 - \gamma_1\lambda_{d1}}\hat{t}_1. \tag{131}$$

An alternative expression for the price index can be obtained by inverting the domestic share λ_{d1} in Definition 2 to obtain

$$P_1 = \left(\frac{N_1^e}{\lambda_{d1}}\right)^{\frac{1}{1-\sigma_1}} \left(\frac{\sigma_1}{\sigma_1 - 1}\right) c_1. \tag{132}$$

Totally differentiating this condition and again using $\hat{c}_1 = (1-\gamma_1)\hat{w} + \gamma_1 \hat{P}_1$, we obtain

$$\hat{P}_1 = \hat{w} + \frac{1}{(1-\gamma_1)(1-\sigma_1)} \left(\hat{N}_1^e - \hat{\lambda}_{d1}\right). \tag{133}$$

Setting (131) equal to (133) we therefore obtain

$$\hat{\lambda}_{d1} = \frac{1-\lambda_{d1}}{1-\gamma_1\lambda_{d1}} \left(-(1-\gamma_1)(\sigma_1 - 1)\hat{w} + (1-\gamma_1)(\sigma_1 - 1)\hat{t}_1 + \hat{N}_1\right). \tag{134}$$

Next, we use the trade balance condition which is written from Definition 2 as

$$\frac{\lambda_{m1} Y_1}{t_1} = N_1^e \left(\frac{\sigma_1}{\sigma_1 - 1}\frac{c_1}{P_1^*}\right)^{1-\sigma_1} Y_1^*.$$

Totally differentiating this condition and using $\hat{c}_1 = (1-\gamma_1)\hat{w} + \gamma_1 \hat{P}_1$ combined again with (133) so that $\hat{c}_1 = \hat{w} + \frac{\gamma_1}{(1-\gamma_1)(1-\sigma_1)}\left(\hat{N}_1^e - \hat{\lambda}_{d1}\right)$, we arrive at

$$\hat{\lambda}_{m1} + \hat{Y}_1 - \hat{t}_1 = \hat{N}_1^e + (1-\sigma_1)\hat{w} + \frac{\gamma_1}{(1-\gamma_1)}\left(\hat{N}_1^e - \hat{\lambda}_{d1}\right). \tag{135}$$

Because $\lambda_{d1} + \lambda_{m1} = 1$ then $\hat{\lambda}_{m1} = -\frac{\lambda_{d1}}{1-\lambda_{d1}}\hat{\lambda}_{d1}$. We also use sector 1 output from Definition 2 which implies that

$$Y_1 = \frac{\alpha_1 (wL + (1-\Lambda_1) Y_1)}{1 - \tilde{\gamma}_1 \Lambda_1} = \frac{\alpha_1}{\alpha_2 + (\alpha_1 - \tilde{\gamma}_1)\Lambda_1} wL,$$

where the second equality is obtained by solving for Y_1 in the first. Totally differentiating this condition, we arrive at

$$\hat{Y}_1 = \hat{w} - \frac{(\alpha_1 - \tilde{\gamma}_1)\Lambda_1}{\alpha_2 + (\alpha_1 - \tilde{\gamma}_1)\Lambda_1}\hat{\Lambda}_1.$$

Substituting this condition and $\hat{\lambda}_{m1} = -\frac{\lambda_{d1}}{1-\lambda_{d1}}\hat{\lambda}_{d1}$ into (135) we obtain

$$-\frac{\lambda_{d1}}{1-\lambda_{d1}}\hat{\lambda}_{d1} - \frac{(\alpha_1 - \tilde{\gamma}_1)\Lambda_1}{\alpha_2 + (\alpha_1 - \tilde{\gamma}_1)\Lambda_1}\hat{\Lambda}_1 - \hat{t}_1 = \hat{N}_1^e - \sigma_1\hat{w} + \frac{\gamma_1}{(1-\gamma_1)}\left(\hat{N}_1^e - \hat{\lambda}_{d1}\right) \tag{136}$$

We substitute $\hat{N}_1^e = \frac{\alpha_2}{\alpha_2+(\alpha_1-\tilde{\gamma}_1)\Lambda_1}\hat{\Lambda}_1$ from Definition 2, define $\mathcal{E}_0 \equiv \left(\frac{\alpha_2+(1-\gamma_1)[(\alpha_1-\tilde{\gamma}_1)\Lambda_1]}{(1-\gamma_1)[\alpha_2+(\alpha_1-\tilde{\gamma}_1)\Lambda_1]}\right)$ and also use $\hat{\Lambda}_1 = \frac{\lambda_{d1}(t_1-1)}{1+\lambda_{d1}(t_1-1)}\hat{\lambda}_{d1} + \frac{\lambda_{d1}-1}{1+\lambda_{d1}(t_1-1)}\hat{t}_1$, to obtain

$$\left[\frac{\gamma_1}{(1-\gamma_1)} - \frac{\lambda_{d1}}{1-\lambda_{d1}} - \frac{\mathcal{E}_0\lambda_{d1}\left(t_1-1\right)}{1+\lambda_{d1}\left(t_1-1\right)}\right]\hat{\lambda}_{d1} = \left[1+\frac{\mathcal{E}_0(\lambda_{d1}-1)}{1+\lambda_{d1}\left(t_1-1\right)}\right]\hat{t}_1 - \sigma_1\hat{w},$$

which is rewritten as

$$\sigma_1\hat{w} = \left[\frac{\lambda_{d1}t_1+(1-\mathcal{E}_0)(1-\lambda_{d1})}{1+\lambda_{d1}\left(t_1-1\right)}\right]\hat{t}_1 + \left[\frac{\lambda_{d1}[1+\mathcal{E}_0\left(t_1-1\right)]+\lambda_{d1}^2\left(t_1-1\right)\left(1-\mathcal{E}_0\right)}{1+\lambda_{d1}\left(t_1-1\right)} - \frac{\gamma_1(1-\lambda_{d1})}{(1-\gamma_1)}\right]\frac{\hat{\lambda}_{d1}}{(1-\lambda_{d1})}.$$

We have therefore obtained two expressions for the change in the domestic share: (134) which is obtained quite directly from its definition, and the above equation that uses trade balance. We eliminate the change in the domestic share from these expressions by substituting (134) into the above equation. We find that:

$$\begin{aligned}
\sigma_1\hat{w} \quad &+ \quad \left[\frac{\lambda_{d1}[1+\mathcal{E}_0\left(t_1-1\right)]+\lambda_{d1}^2\left(t_1-1\right)\left(1-\mathcal{E}_0\right)}{1+\lambda_{d1}\left(t_1-1\right)} - \frac{\gamma_1(1-\lambda_{d1})}{(1-\gamma_1)}\right]\frac{\left(1-\gamma_1\right)\left(\sigma_1-1\right)\hat{w}}{1-\gamma_1\lambda_{d1}} \\
&= \quad \left[\frac{\lambda_{d1}[1+\mathcal{E}_0\left(t_1-1\right)]+\lambda_{d1}^2\left(t_1-1\right)\left(1-\mathcal{E}_0\right)}{1+\lambda_{d1}\left(t_1-1\right)} - \frac{\gamma_1(1-\lambda_{d1})}{(1-\gamma_1)}\right]\frac{\left(1-\gamma_1\right)\left(\sigma_1-1\right)}{1-\gamma_1\lambda_{d1}}\hat{t}_1 \\
&+ \quad \left[\frac{\lambda_{d1}t_1+(1-\mathcal{E}_0)(1-\lambda_{d1})}{1+\lambda_{d1}\left(t_1-1\right)}\right]\hat{t}_1 \\
&+ \quad \left[\frac{\lambda_{d1}[1+\mathcal{E}_0\left(t_1-1\right)]+\lambda_{d1}^2\left(t_1-1\right)\left(1-\mathcal{E}_0\right)}{1+\lambda_{d1}\left(t_1-1\right)} - \frac{\gamma_1(1-\lambda_{d1})}{(1-\gamma_1)}\right]\frac{\hat{N}_1^e}{1-\gamma_1\lambda_{d1}}.
\end{aligned}$$

This equation is simplified by using:

$$\left[\frac{\lambda_{d1}[1+\mathcal{E}_0(t_1-1)]+\lambda_{d1}^2(t_1-1)(1-\mathcal{E}_0)}{1+\lambda_{d1}(t_1-1)} - \frac{\gamma_1(1-\lambda_{d1})}{(1-\gamma_1)}\right] = \left[\frac{\lambda_{d1}t_1\mathcal{E}_0}{1+\lambda_{d1}(t_1-1)} + \lambda_{d1}(1-\mathcal{E}_0) + 1 - \frac{(1-\gamma_1)\lambda_{d1}}{(1-\gamma_1)}\right].$$

Substituting this above, we find after some simplification that

$$\hat{w} = \mathcal{E}_1^{hom}\hat{t}_1 + \mathcal{E}_2^{hom}\hat{N}_1^e, \text{ where}$$

$$
\begin{aligned}
\mathcal{E}_1^{hom} &\equiv \frac{\left[\frac{\lambda_{d1}t_1\mathcal{E}_0}{1+\lambda_{d1}(t_1-1)}+\lambda_{d1}(1-\mathcal{E}_0)+1\right]\frac{(1-\gamma_1)(\sigma_1-1)}{1-\gamma_1\lambda_{d1}}-(\sigma_i-1)+\left[\frac{\lambda_{d1}t_1+(1-\mathcal{E}_0)(1-\lambda_{d1})}{1+\lambda_{d1}(t_1-1)}\right]}{\left[\frac{\lambda_{d1}t_1\mathcal{E}_0}{1+\lambda_{d1}(t_1-1)}+\lambda_{d1}(1-\mathcal{E}_0)+1\right]\frac{(1-\gamma_1)(\sigma_1-1)}{1-\gamma_1\lambda_{d1}}+1}. \\
\mathcal{E}_2^{hom} &\equiv \frac{1}{(1-\gamma_1)(\sigma_1-1)}\frac{\left[\frac{\lambda_{d1}t_1\mathcal{E}_0}{1+\lambda_{d1}(t_1-1)}+\lambda_{d1}(1-\mathcal{E}_0)+1\right]\frac{(1-\gamma_1)(\sigma_1-1)}{1-\gamma_1\lambda_{d1}}-(\sigma_i-1)}{\left[\frac{\lambda_{d1}t_1\mathcal{E}_0}{1+\lambda_{d1}(t_1-1)}+\lambda_{d1}(1-\mathcal{E}_0)+1\right]\frac{(1-\gamma_1)(\sigma_1-1)}{1-\gamma_1\lambda_{d1}}+1}.
\end{aligned}
$$

Noting that when $\alpha = 1$ then $\mathcal{E}_0 = 1$, we obtain the following result in the one-sector model when $t_1 = 1$, where we now add the argument to $\mathcal{E}_1^{hom}(\gamma_1)$ and the supercript to σ_1^{hom}:

$$
\mathcal{E}_1^{hom}(\gamma_1) = \frac{1-\frac{\left(\sigma_1^{hom}-2\right)}{\sigma_1^{hom}}\frac{\gamma_1(1-\lambda_{d1})}{(1-\gamma_1)\lambda_{d1}}}{\frac{1+\lambda_{d1}}{\lambda_{d1}}-\frac{1}{\sigma_1^{hom}}+\frac{\gamma_1(1-\lambda_{d1})}{(1-\gamma_1)}\frac{1}{\sigma_1^{hom}\lambda_{d1}}}. \tag{137}
$$

It is immediate that $\mathcal{E}_1^{hom}(0) > 0$. We also see that $\mathcal{E}_1^{hom}(\gamma_1) < 0$ if and only if

$$
1 < \frac{\left(\sigma_1^{hom}-2\right)}{\sigma_1^{hom}}\frac{\gamma_1(1-\lambda_{d1})}{(1-\gamma_1)\lambda_{d1}} \iff \eta_{m1} > \frac{\sigma_1^{hom}}{\left(\sigma_1^{hom}-2\right)}. \tag{138}
$$

Because η_{m1} is increasing in γ_1, we confirm that $\mathcal{E}_1^{hom}(\gamma_1)$ is declining in γ_1 by checking its derivative with respect to η_{m1}:

$$
\begin{aligned}
\frac{d\mathcal{E}_1^{hom}}{d\eta_{m1}} &= -\frac{\left(\frac{\sigma_1^{hom}-2}{\sigma_1^{hom}}\right)\left[\frac{1+\lambda_{d1}}{\lambda_{d1}}+(\eta_{m1}-1)\frac{1}{\sigma_1^{hom}}\right]+\frac{1}{\sigma_1^{hom}}\left[1-\eta_{m1}\left(\frac{\sigma_1^{hom}-2}{\sigma_1^{hom}}\right)\right]}{\left[\frac{1+\lambda_{d1}}{\lambda_{d1}}+(\eta_{m1}-1)\frac{1}{\sigma_1^{hom}}\right]^2} \\
&= -\frac{\left(\frac{\sigma_1^{hom}-2}{\sigma_1^{hom}}\right)\frac{1+\lambda_{d1}}{\lambda_{d1}}-\frac{1}{\sigma_1^{hom}}\left(\frac{\sigma_1^{hom}-2}{\sigma_1^{hom}}\right)+\frac{1}{\sigma_1^{hom}}}{\left[\frac{1+\lambda_{d1}}{\lambda_{d1}}+(\eta_{m1}-1)\frac{1}{\sigma_1^{hom}}\right]^2} = -\frac{\left(\frac{\sigma_1^{hom}-2}{\sigma_1^{hom}}\right)\frac{1+\lambda_{d1}}{\lambda_{d1}}+\frac{2}{(\sigma_1^{hom})^2}}{\left[\frac{1+\lambda_{d1}}{\lambda_{d1}}+(\eta_{m1}-1)\frac{1}{\sigma_1^{hom}}\right]^2},
\end{aligned}
$$

which is negative provided that $\sigma_1^{hom} > 2$, which proves the statements about $\mathcal{E}_1^{hom}(\gamma_1)$ in (30).

Finally, we want compare $\mathcal{E}_1^{het}(0)$ in (129) to $\mathcal{E}_1^{hom}(0)$ in (137), for given λ_{d1}. The inequality in (31) follows because the denominators of (129) and (137) are equal and

$$
-\frac{\rho_1^{hom}}{\theta_1} < -\frac{\rho_1^{het}}{\theta_1},
$$

since $\sigma_1^{hom} - 1 = \theta_1 > \sigma_1^{het} - 1 \iff \rho_1^{hom} > \rho_1^{het}$.

D.2 Preliminary Change in Utility

We obtain here a preliminary expression for the change in indirect utility, which is (up to a constant):

$$U = \frac{wL + B}{P_1^{\alpha_1} P_2^{\alpha_2}}. \tag{139}$$

The price index in sector 2 in similar to that shown in (104), except that $\lambda_{d2} \equiv 1$. Using these in (139), we obtain:

$$\hat{U} = \frac{B}{wL+B}(\hat{B} - \hat{w}) - \frac{\alpha_1}{\theta_1(1-\gamma_1)}\hat{\lambda}_{d1} + \sum_{s=1,2} \frac{\alpha_s}{\theta_s(1-\gamma_s)} \left[\hat{N}_s^e + \left(\frac{\theta_s}{\sigma_s - 1} - 1\right)(\hat{Y}_s - \hat{w})\right]. \tag{140}$$

Note that the market clearing condition (4) can be rewritten as

$$Y_1 = \alpha_1(wL+B) + \tilde{\gamma}_1 \left(E_{d1} + E_{x1}\right) = \alpha_1(wL+B) + w\,\tilde{\gamma}_1 f_1^e \left(\frac{\theta_1\,\sigma_1}{\sigma_1 - 1}\right) N_1^e,$$

where we have used expenditure on the differentiated input in sector 1, $E_{d1} + E_{x1} = \lambda_{d1} Y_1 + \lambda_{x1} Y_1^*$, and also entry from (78). Expenditure on the differentiated input from home and foreign firms equals the total expenditure on that industry, which we write as $E_1 \equiv E_{d1} + E_{x1}$. It follows that $Y_s = \alpha_s(wL+B) + \tilde{\gamma}_s E_s$, with $E_2 = Y_2$ because the finished good in the nontraded sector is bundled together from the domestically-produced intermediate inputs. Totally differentiating we obtain

$$\hat{Y}_s - \hat{w} = \frac{\alpha_s B}{Y_s}(\hat{B} - \hat{w}) + \left(\frac{\tilde{\gamma}_s E_s}{Y_s}\right)\hat{N}_s^e.$$

Substituting into (140), we have

$$\begin{aligned}
\hat{U} &= \frac{B}{wL+B}(\hat{B} - \hat{w}) - \frac{\alpha_1}{\theta_1(1-\gamma_1)}\hat{\lambda}_{d1} \\
&+ \sum_{s=1,2} \frac{\alpha_s}{\theta_s(1-\gamma_s)} \left[\hat{N}_s^e + \left(\frac{\theta_s}{\sigma_s - 1} - 1\right)\left(\frac{B}{Y_s}(\hat{B} - \hat{w}) + \left(\frac{\tilde{\gamma}_s \Gamma_s}{Y_s}\right)\hat{N}_s^e\right)\right] \\
&= -\frac{\alpha_1}{\theta_1(1-\gamma_1)}\hat{\lambda}_{d1} + \sum_{s=1,2} \alpha_s \left[1 + \frac{\alpha_s(wL+B)}{Y_s\theta_s(1-\gamma_s)}\left(\frac{\theta_s}{\sigma_s - 1} - 1\right)\right] \frac{B}{wL+B}(\hat{B} - \hat{w}) \\
&+ \sum_{s=1,2} \frac{\alpha_s}{\theta_s(1-\gamma_s)} \left[\left(1 - \frac{\tilde{\gamma}_s E_s}{Y_s}\right) + \left(\frac{\theta_s}{\sigma_s - 1}\right)\left(\frac{\tilde{\gamma}_s E_s}{Y_s}\right)\right]\hat{N}_s^e,
\end{aligned}$$

which appears as (32) once we define $\Gamma_s \equiv \tilde{\gamma}_s E_s / Y_s$ as the fraction of finished output in sector s that is used as an intermediate input, with $\Gamma_2 = \tilde{\gamma}_2$ because $E_2 = Y_2$.

We also derive the similar expression but with homogeneous firms. In that case the sector 1 price index is given by (23), and likewise in sector except with $\lambda_{d12} \equiv 1$. Denoting the elasticities of substitution in these expressions by σ_s^{hom} for $s, 1, 2$, we use the price indexes in (139) to obtain

$$\hat{U} = \frac{B}{wL + B}(\hat{B} - \hat{w}) - \frac{\alpha_1}{(\sigma_1^{hom} - 1)(1 - \gamma_1)}\hat{\lambda}_{d1} + \sum_{s=1,2} \frac{\alpha_s}{(\sigma_s^{hom} - 1)(1 - \gamma_s)}\hat{N}_s^e,$$

which justifies (33) in the main text. Finally, from the labor market clearing condition $L_1 + L_2 = L$ and using (91) and (92), we have the result reported in note 17:

$$0 = \frac{L_2}{L}\hat{N}_2^e + \frac{L_1}{L}\hat{N}_1^e \implies \hat{N}_2^e = -\frac{L_1}{L_2}\hat{N}_1^e. \tag{141}$$

D.3 Total Change in Utility

Focusing for the remainder of the Appendix on the heterogeneous firm model, we drop the superscript from σ_1^{het} and substitute $\mathcal{E}_n^{het}$, $n = 1, 2$ into (125), to obtain

$$\begin{aligned}
\hat{\varphi}_{x1} &= \frac{\tilde{\lambda}_{d1}}{1 + \tilde{\lambda}_{d1}} \frac{1}{\theta_1} \left(1 + \mathcal{E}_2^{het}\right) \hat{N}_1^e + \frac{\tilde{\lambda}_{d1}}{1 + \tilde{\lambda}_{d1}} \left(\frac{\mathcal{E}_1^{het}}{\theta_1} + \frac{1}{\rho_1}\right) \hat{t}_1 \\
&= \frac{\tilde{\lambda}_{d1}}{1 + \tilde{\lambda}_{d1}} \left(\frac{\frac{1}{\rho_1}\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}} + \frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{1-\lambda_{d1}\tilde{\lambda}_{d1}}{1-\tilde{\lambda}_{d1}} + (t_1 - 1)\lambda_{d1}\right)}{\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}} - \frac{\rho_1}{\theta_1} + \frac{\gamma_1}{(1-\gamma_1)}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{\rho_1}{\theta_1} + \frac{1}{\sigma_1}(t_1 - 1)\lambda_{d1}\right)} \right) \hat{t}_1 \\
&+ \frac{\tilde{\lambda}_{d1}}{1 + \tilde{\lambda}_{d1}} \frac{1}{\theta_1} \left(1 + \mathcal{E}_2^{het}\right) \hat{N}_1^e.
\end{aligned} \tag{142}$$

Note that: $$\frac{\tilde{\lambda}_{d1}}{1 + \tilde{\lambda}_{d1}} \frac{1}{\theta_1} \left(1 + \mathcal{E}_2^{het}\right) = \frac{\frac{1}{\theta_1}\frac{1+(1-\gamma_1)(\sigma_1-1)}{\sigma_1(1-\gamma_1)}}{\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}} - \frac{\rho_1}{\theta_1} + \frac{\gamma_1}{(1-\gamma_1)}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{\rho_1}{\theta_1} - \frac{1}{\sigma_1}(1 - t_1)\lambda_{d1}\right)}.$$

Then from the welfare equation (119), using (120) we obtain

$$\begin{aligned}
\hat{U} &= \alpha_1 \left[\mathcal{E}_3 (1 - \lambda_{d1}) \hat{t}_1 + \left(\mathcal{E}_3 \theta_1 \left(\frac{\lambda_{d1} - \tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}} \right) - \frac{1}{(1 - \gamma_1)} \frac{(1 - \tilde{\lambda}_{d1})}{\tilde{\lambda}_{d1}} \right) \hat{\varphi}_{x1} \right] \\
&+ \alpha_1 \frac{1 + (1 - \gamma_1)(\sigma_1 - 1)}{(1 - \gamma_1)(\sigma_1 - 1)} \hat{N}_1^e + (\alpha_2) \frac{1 + (1 - \gamma_2)(\sigma_2 - 1)}{(1 - \gamma_2)(\sigma_2 - 1)} \hat{N}_2^e,
\end{aligned} \tag{143}$$

$$\text{with} \quad \mathcal{E}_3 \equiv \left(1 + \frac{\gamma_1(\sigma_1 - 1)}{(1 + (1-\gamma_1)(\sigma_1 - 1)) + \sigma_1(t_1 - 1)(1 - \tilde{\lambda}_{d1})} + \frac{1}{(1-\gamma_1)(\sigma_1 - 1)}\right). \tag{144}$$

Inverting (142), $\hat{t}_1$ is given by

$$\begin{aligned}
\hat{t}_1 &= -\frac{\frac{1}{\theta_1}\frac{1+(1-\gamma_1)(\sigma_1-1)}{\sigma_1(1-\gamma_1)}}{\frac{\tilde{\lambda}_{d1}}{1+\tilde{\lambda}_{d1}}\left(\frac{1}{\rho_1}\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}} + \frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{1-\lambda_{d1}\tilde{\lambda}_{d1}}{1-\tilde{\lambda}_{d1}} + (t_1 - 1)\lambda_{d1}\right)\right)}\hat{N}_1^e \\
&+ \frac{\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}} - \frac{\rho_1}{\theta_1} + \frac{\gamma_1}{(1-\gamma_1)}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{\rho_1}{\theta_1} + \frac{1}{\sigma_1}(t_1 - 1)\lambda_{d1}\right)}{\frac{\tilde{\lambda}_{d1}}{1+\tilde{\lambda}_{d1}}\left(\frac{1}{\rho_1}\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}} + \frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{1-\lambda_{d1}\tilde{\lambda}_{d1}}{1-\tilde{\lambda}_{d1}} + (t_1 - 1)\lambda_{d1}\right)\right)}\hat{\varphi}_{x1}.
\end{aligned}$$

Write this expression for $\hat{t}_1$ as

$$\hat{t}_1 = -\mathcal{E}_4\hat{N}_1^e + \mathcal{E}_5\hat{\varphi}_{x1}, \tag{145}$$

where

$$\mathcal{E}_4 = \frac{\frac{1}{\theta_1}\frac{1+(1-\gamma_1)(\sigma_1-1)}{\sigma_1(1-\gamma_1)}}{\frac{\tilde{\lambda}_{d1}}{1+\tilde{\lambda}_{d1}}\left(\frac{1}{\rho_1}\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}} + \frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{1-\lambda_{d1}\tilde{\lambda}_{d1}}{1-\tilde{\lambda}_{d1}} + (t_1 - 1)\lambda_{d1}\right)\right)},$$

$$\mathcal{E}_5 = \frac{\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}} - \frac{\rho_1}{\theta_1} + \frac{\gamma_1}{(1-\gamma_1)}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{\rho_1}{\theta_1} + \frac{1}{\sigma_1}(t_1 - 1)\lambda_{d1}\right)}{\frac{\tilde{\lambda}_{d1}}{1+\tilde{\lambda}_{d1}}\left(\frac{1}{\rho_1}\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}} + \frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{1-\lambda_{d1}\tilde{\lambda}_{d1}}{1-\tilde{\lambda}_{d1}} + (t_1 - 1)\lambda_{d1}\right)\right)}.$$

Using (85), $\mathcal{E}_4$ and $\mathcal{E}_5$ can be written as

$$\mathcal{E}_4 = \frac{\frac{1}{\theta_1}\frac{1+(1-\gamma_1)(\sigma_1-1)}{\sigma_1(1-\gamma_1)}}{\frac{1}{\rho_1} + \frac{1}{\theta_1}\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\sigma_1}(1-\lambda_{d1})}, \tag{146}$$

$$\mathcal{E}_5 = \frac{\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}} - \frac{\rho_1}{\theta_1} + \frac{\gamma_1}{(1-\gamma_1)}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{\rho_1}{\theta_1} + \frac{1}{\sigma_1}(t_1 - 1)\lambda_{d1}\right)}{\frac{1}{\rho_1} + \frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{d1})}. \tag{147}$$

Now we simplify the welfare expression in (143). First, note that using (145), we obtain

$$\begin{aligned}
\hat{U} &= \alpha_1\left[\mathcal{E}_3(1-\lambda_{d1})\mathcal{E}_5 + \mathcal{E}_3\theta_1\left(\frac{\lambda_{d1} - \tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\right) - \frac{1}{(1-\gamma_1)}\frac{(1-\tilde{\lambda}_{d1})}{\tilde{\lambda}_{d1}}\right]\hat{\varphi}_{x1} \\
&\quad + \alpha_1\left[\frac{1+(1-\gamma_1)(\sigma_1 - 1)}{1-\gamma_1(\sigma_1 - 1)} - \mathcal{E}_3(1-\lambda_{d1})\mathcal{E}_4\right]\hat{N}_1^e + \alpha_2\frac{1+(1-\gamma_2)(\sigma_2-1)}{(1-\gamma_2)(\sigma_2 - 1)}\hat{N}_2^e. \qquad (148)
\end{aligned}$$

We seek to express $\hat{N}_1^e$ and $\hat{N}_2^e$ as functions of $\hat{\varphi}_{x1}$. Recalling (91), (92) and (96), we obtain

$$N_2^e w f_2^e \left(\frac{\theta_2\sigma_2}{\sigma_2 - 1}\right) = \frac{1}{1-\tilde{\gamma}_2} wL_2 = \frac{1}{1-\tilde{\gamma}_2} wL \left(\frac{(1-\tilde{\lambda}_{d1})t_1 + \tilde{\lambda}_{d1} - \tilde{\gamma}_1}{\left(1-\tilde{\lambda}_{d1}\right)t_1 + \tilde{\lambda}_{d1} + \frac{1}{\alpha_2}\left(\alpha_1 - \tilde{\gamma}_1\right)}\right).$$

Here we define

$$l_2 \equiv \frac{(1-\tilde{\lambda}_{d1})t_1 + \tilde{\lambda}_{d1} - \tilde{\gamma}_1}{(1-\tilde{\lambda}_{d1})t_1 + \tilde{\lambda}_{d1} + \frac{1}{\alpha_2}\left(\alpha_1 - \tilde{\gamma}_1\right)} = \frac{L_2}{L},$$

and then $\hat{N}_2^e$ is given by

$$\hat{N}_2^e = \hat{l}_2,$$

where

$$\hat{l}_2 = (1-l_2)\frac{(1-t_1)\,\tilde{\lambda}_{d1}\hat{\tilde{\lambda}}_{d1} + \left(1-\tilde{\lambda}_{d1}\right)t_1\hat{t}_1}{(1-\tilde{\lambda}_{d1})t_1 + \tilde{\lambda}_{d1} - \tilde{\gamma}_1}.$$

Combining this expression with (141), $\hat{N}_1^e$ can be written as

$$\begin{aligned}\hat{N}_1^e &= -\frac{L_2}{L_1}(1-l_2)\left(\frac{(1-t_1)\,\tilde{\lambda}_{d1}\hat{\tilde{\lambda}}_{d1} + (1-\tilde{\lambda}_{d1})t_1\hat{t}_1}{\left(1-\tilde{\lambda}_{d1}\right)t_1 + \tilde{\lambda}_{d1} - \tilde{\gamma}_1}\right)\\ &= -\left(\frac{(1-t_1)\,\tilde{\lambda}_{d1}\hat{\tilde{\lambda}}_{d1} + (1-\tilde{\lambda}_{d1})t_1\hat{t}_1}{(1-\tilde{\lambda}_{d1})t_1 + \tilde{\lambda}_{d1} + \frac{1}{\alpha_2}\left(\alpha_1 - \tilde{\gamma}_1\right)}\right).\end{aligned}$$

From (82) and (116), we can use

$$\hat{\tilde{\lambda}}_{d1} = \theta_1\frac{\left(1-\tilde{\lambda}_{d1}\right)}{\tilde{\lambda}_{d1}}\hat{\varphi}_{x1}, \tag{149}$$

and combining with (145), we obtain

$$\begin{aligned}&-\left((1-\tilde{\lambda}_{d1})t_1 + \tilde{\lambda}_{d1} + \frac{1}{\alpha_2}\left(\alpha_1 - \tilde{\gamma}_1\right)\right)\hat{N}_1^e\\ &= (1-t_1)\,\tilde{\lambda}_{d1}\theta_1\frac{\left(1-\tilde{\lambda}_{d1}\right)}{\tilde{\lambda}_{d1}}\;\hat{\varphi}_{x1} + (1-\tilde{\lambda}_{d1})t_1(\mathcal{E}_5\hat{\varphi}_{x1} - \mathcal{E}_4\hat{N}_1^e).\end{aligned}$$

Then we arrive at

$$\hat{N}_1^e = \frac{(1-\tilde{\lambda}_{d1})\left((1-t_1)\,\theta_1 + t_1\mathcal{E}_5\right)}{\left(\left(1-\tilde{\lambda}_{d1}\right)t_1\left(\mathcal{E}_4 - 1\right) - \tilde{\lambda}_{d1} - \frac{1}{\alpha_2}\left(\alpha_1 - \tilde{\gamma}_1\right)\right)}\hat{\varphi}_{x1}. \tag{150}$$

Finally, from (96), (97) and (141), $\hat{N}_2^e$ can be written as

$$\hat{N}_2^e = -\left(\frac{\alpha_1}{\alpha_2}\right)\frac{(1-\tilde{\gamma}_1)}{(1-\tilde{\lambda}_{d1})t_1+\tilde{\lambda}_{d1}-\tilde{\gamma}_1}\hat{N}_1^e. \tag{151}$$

D.4 Definitions of $D(t_1)$ and $\mathcal{E}_\varphi$

We can use the above equations to obtain the total change in utility. Substituting (150) and (151) into the second term of welfare in (148), we have

$$\alpha_1\left(\frac{1+(1-\gamma_1)(\sigma_1-1)}{1-\gamma_1(\sigma_1-1)}-\mathcal{E}_3(1-\lambda_{d1})\mathcal{E}_4\right)\hat{N}_1^e+\alpha_2\frac{\sigma_2}{\sigma_2-1}\hat{N}_2^e = D(t_1)\alpha_1\hat{N}_1^e,$$

where

$$D(t_1)\equiv\frac{1+(1-\gamma_1)(\sigma_1-1)}{(1-\gamma_1)(\sigma_1-1)}-\frac{1-\tilde{\gamma}_2}{(1-\gamma_2)\rho_2}\frac{1+(1-\gamma_1)(\sigma_1-1)}{\sigma_1\left(1-\tilde{\lambda}_{d1}\right)(t_1-1)+(1+(1-\gamma_1)(\sigma_1-1))}-\mathcal{E}_3\mathcal{E}_4(1-\lambda_{d1}).$$

This initial definition $D(t_1)$ can be re-expressed using the function $T(t_1)$ in (86) to obtain the alternative definition

$$D(t_1)\equiv\left[\frac{1+(1-\gamma_1)(\sigma_1-1)}{(1-\gamma_1)(\sigma_1-1)}-\left(\frac{1+(1-\gamma_2)(\sigma_2-1)}{(1-\gamma_2)(\sigma_2-1)}\right)\frac{(1-\tilde{\gamma}_1)}{T(t_1)}-\mathcal{E}_3\mathcal{E}_4(1-\lambda_{d1})\right]. \tag{152}$$

Notice that the definition of $D(t_1)$ used in the main text, is obtained by further defining

$$\mathcal{E}_d\equiv\mathcal{E}_3\mathcal{E}_4(1-\lambda_{d1}), \tag{153}$$

and using (87) to derive $\frac{(1-\tilde{\gamma}_1)}{T(t_1)}=\frac{\Lambda_1(1-\tilde{\gamma}_1)}{1-\tilde{\gamma}_1\Lambda_1}$, and also using the effective distortions $\frac{\tilde{\sigma}_s}{(\tilde{\sigma}_s-1)}$ defined in (16) so that expression (35) in the main text follows.

It follows that $\hat{U}$ can be written as shown in (34) in the main text,

$$\hat{U}=\alpha_1\left[\mathcal{E}_\varphi\hat{\varphi}_{x1}+D(t_1)\hat{N}_1^e\right],$$

where

$$\mathcal{E}_{\varphi} \equiv \mathcal{E}_3 \left(1-\lambda_{d1}\right) \mathcal{E}_5 + \mathcal{E}_3 \theta_1 \left(\frac{\lambda_{d1}-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\right) - \left(\frac{1-\tilde{\lambda}_{d1}}{(1-\gamma_1)\tilde{\lambda}_{d1}}\right). \tag{154}$$

We see that the total change in utility in (34) is written as the sum of two terms: the first given by $\alpha_1 \mathcal{E}_{\varphi} \hat{\varphi}_{x1}$ reflects selection and includes all the changes in cutoff productivities; and second $\alpha_1 D(t_1) \hat{N}_1^e$ reflects entry. At the optimum, $\hat{U}/\left(\alpha_1 \hat{\varphi}_{x1}\right) = 0$, which implies from (150) that

$$\begin{aligned} &\left[\mathcal{E}_3 \left(1-\lambda_{d1}\right) \mathcal{E}_5 + \mathcal{E}_3 \theta_1 \left(\frac{\lambda_{d1}-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\right) - \frac{1}{(1-\gamma_1)} \frac{\left(1-\tilde{\lambda}_{d1}\right)}{\tilde{\lambda}_{d1}}\right] \\ &= -D(t_1) \frac{(1-\tilde{\lambda}_{d1})\left(\left(1-t_1\right)\theta_1 + t_1 \mathcal{E}_5\right)}{\left(1-\tilde{\lambda}_{d1}\right) t_1 \left(\mathcal{E}_4 - 1\right) - \tilde{\lambda}_{d1} - \frac{1}{\alpha_2}\left(\alpha_1 - \tilde{\gamma}_1\right)}. \end{aligned} \tag{155}$$

Using the tariff formula (84) repeatedly, we define

$$\tilde{M}(t_1) = \frac{(1-\gamma_1)\left(\mathcal{E}_5 - \frac{(t_1-1)}{t_1}\theta_1\right)}{\alpha_2\left(\left(1-\tilde{\lambda}_{d1}\right) t_1 \left(1-\mathcal{E}_4\right) + \tilde{\lambda}_{d1}\right) + \alpha_1 - \tilde{\gamma}_1} \frac{\tilde{\lambda}_{d1}}{\lambda_{d1}} D(t_1), \tag{156}$$

and then the first-order condition (155) becomes

$$\left[\mathcal{E}_3 \left(1-\lambda_{d1}\right) \mathcal{E}_5 + \mathcal{E}_3 \theta_1 \left(\frac{\lambda_{d1}-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\right) - \frac{1}{(1-\gamma_1)} \frac{\left(1-\tilde{\lambda}_{d1}\right)}{\tilde{\lambda}_{d1}}\right] = \frac{\left(1-\lambda_{d1}\right)\alpha_2}{(1-\gamma_1)} \tilde{M}(t_1).$$

Using $\frac{1-t_1}{t_1}\left(1-\lambda_{d1}\right) = \frac{\lambda_{d1}-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}$ from (84), we get

$$\frac{\left(1-\lambda_{d1}\right)}{(1-\gamma_1)} \left[(1-\gamma_1)\mathcal{E}_3\mathcal{E}_5 + (1-\gamma_1)\mathcal{E}_3\theta_1 \left(\frac{1-t_1}{t_1}\right) - \frac{1}{\lambda_{d1}t_1}\right] = \frac{\left(1-\lambda_{d1}\right)\alpha_2}{(1-\gamma_1)} \tilde{M}(t_1),$$

$$\mathcal{E}_5 + \theta_1 \left(\frac{1-t_1}{t_1}\right) - \frac{1}{\lambda_{d1}t_1(1-\gamma_1)\mathcal{E}_3} = \alpha_2 \frac{\tilde{M}(t_1)}{(1-\gamma_1)\mathcal{E}_3}.$$

Using (147), we obtain

$$\begin{aligned} &\left(\frac{1-t_1}{t_1}\right)\frac{\theta_1}{\rho_1} + \frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}} - \frac{\rho_1}{\theta_1} + \frac{\gamma_1}{(1-\gamma_1)} \frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}} \frac{\rho_1}{\theta_1} - \frac{\frac{1}{\rho_1} + \frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{d1}\right)}{\lambda_{d1}t_1(1-\gamma_1)\mathcal{E}_3} \\ &= \alpha_2 \frac{\tilde{M}(t_1)}{(1-\gamma_1)\mathcal{E}_3} \left(\frac{1}{\rho_1} + \frac{\gamma_1}{(1-\gamma_1)} \frac{1}{\theta_1} \frac{1}{\sigma_1}\left(1-\lambda_{d1}\right)\right). \end{aligned}$$

We multiply both sides by t_1 and use (84) again to get

$$
\begin{aligned}
&(1-t_1)\frac{\theta_1}{\rho_1}+\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}t_1-\frac{\rho_1}{\theta_1}t_1+\frac{\gamma_1}{(1-\gamma_1)}\frac{(1-\lambda_{d1})}{\lambda_{d1}}\frac{\rho_1}{\theta_1}-\frac{\frac{1}{\rho_1}+\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{d1})}{\lambda_{d1}(1-\gamma_1)\mathcal{E}_3}\\
&=\alpha_2\frac{\tilde{M}(t_1)t_1}{(1-\gamma_1)\mathcal{E}_3}\left(\frac{1}{\rho_1}+\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{d1})\right).
\end{aligned}
$$

Next we add and subtract $\frac{1}{\lambda_{d1}}$ and use $\frac{t_1}{\tilde{\lambda}_{d1}}=t_1-1+\frac{1}{\lambda_{d1}}$, to obtain

$$
\begin{aligned}
&(1-t_1)\left(\frac{\theta_1-\rho_1}{\rho_1}\right)+t_1-\frac{\rho_1}{\theta_1}t_1+\frac{\gamma_1}{(1-\gamma_1)}\frac{(1-\lambda_{d1})}{\lambda_{d1}}\frac{\rho_1}{\theta_1}-\left(\frac{\frac{1}{\rho_1}+\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{d1})}{(1-\gamma_1)\mathcal{E}_3}-1\right)\frac{1}{\lambda_{d1}}\\
=\;&\alpha_2\frac{\tilde{M}(t_1)t_1}{(1-\gamma_1)\mathcal{E}_3}\left(\frac{1}{\rho_1}+\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{d1})\right).
\end{aligned}
$$

Note that

$$
\frac{\frac{1}{\rho_1}+\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{d1})}{(1-\gamma_1)\mathcal{E}_3}-1=\gamma_1\frac{\frac{1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{d1})+\frac{\rho_1+(1-t_1)(1-\tilde{\lambda}_{d1})-1}{\gamma_1\rho_1+(1-t_1)\left(1-\tilde{\lambda}_{d1}\right)-1}}{(1-\gamma_1)\mathcal{E}_3}.
$$

Then the first-order condition becomes

$$
\begin{aligned}
&\left(\frac{\theta_1}{\theta_1-\rho_1}-t_1\right)\frac{(\theta_1-\rho_1)^2}{\theta_1\rho_1}+\frac{\gamma_1}{(1-\gamma_1)}\frac{(1-\lambda_{d1})}{\lambda_{d1}}\frac{\rho_1}{\theta_1}-\gamma_1\frac{\frac{1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{d1})+\frac{\rho_1+(1-t_1)\left(1-\tilde{\lambda}_{d1}\right)-1}{\gamma_1\rho_1+(1-t_1)(1-\tilde{\lambda}_{d1})-1}}{(1-\gamma_1)\mathcal{E}_3}\frac{1}{\lambda_{d1}}\\
&=\alpha_2\frac{\tilde{M}(t_1)t_1}{(1-\gamma_1)\mathcal{E}_3}\left(\frac{1}{\rho_1}+\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{d1})\right).
\end{aligned}
$$

Now we find the common denominator for the second terms on the left-hand side using (144):

$$
\begin{aligned}
&\frac{\gamma_1}{(1-\gamma_1)}\frac{(1-\lambda_{d1})}{\lambda_{d1}}\frac{\rho_1}{\theta_1}-\gamma_1\frac{\frac{1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{d1})+\frac{\rho_1+(1-t_1)\left(1-\tilde{\lambda}_{d1}\right)-1}{\gamma_1\rho_1+(1-t_1)(1-\tilde{\lambda}_{d1})-1}}{(1-\gamma_1)\mathcal{E}_3}\frac{1}{\lambda_{d1}}\\
&=\frac{\gamma_1}{(1-\gamma_1)\mathcal{E}_3}\rho_1\left(\frac{(1-\lambda_{d1})}{\lambda_{d1}}\frac{\mathcal{E}_3}{\theta_1}-\frac{\frac{1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{d1})+\frac{\rho_1+(1-t_1)(1-\tilde{\lambda}_{d1})-1}{\gamma_1\rho_1+(1-t_1)\left(1-\tilde{\lambda}_{d1}\right)-1}}{\rho_1}\frac{1}{\lambda_{d1}}\right)\\
&=\frac{\gamma_1}{(1-\gamma_1)\mathcal{E}_3}\frac{1}{\theta_1}\frac{1}{\lambda_{d1}}\frac{\theta_1-\theta_1\rho_1-(1-\lambda_{d1})\rho_1+(\theta_1-\rho_1(1-\lambda_{d1}))(t_1-1)\left(1-\tilde{\lambda}_{d1}\right)}{\gamma_1\rho_1+(1-t_1)(1-\tilde{\lambda}_{d1})-1}.
\end{aligned}
$$

Then the first-order condition becomes

$$\left(\frac{\theta_1}{\theta_1-\rho_1}-t_1\right)\frac{(\theta_1-\rho_1)^2}{\theta_1\rho_1}-\frac{(\gamma_1)}{(1-\gamma_1)\mathcal{E}_3}\frac{\tilde{R}(t_1)}{\theta_1}=\alpha_2\frac{\tilde{M}(t_1)t_1}{(1-\gamma_1)\mathcal{E}_3}\left(\frac{1}{\rho_1}+\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{d1})\right),$$

$$\text{with }\ \tilde{R}(t_1)\equiv\frac{1}{\lambda_{d1}}\left(\frac{\theta_1(1-\rho_1)-\rho_1(1-\lambda_{d1})+(\theta_1-\rho_1(1-\lambda_{d1}))(t_1-1)(1-\tilde{\lambda}_{d1})}{(1-\gamma_1)\rho_1+(t_1-1)\left(1-\tilde{\lambda}_{d1}\right)}\right). \tag{157}$$

The first-order condition for t_1^* can then be written succinctly using $t^{het}=\frac{\theta_1}{\theta_1-\rho_1}$ as

$$t^{het}[1-\gamma_1 R(t_1^*)]=t_1^*[1+\alpha_2 M(t_1^*)], \tag{158}$$

where we define the terms $M(t_1)$ and $R(t_1)$ as in the following subsections. Dividing through by $[1+\alpha_2 M(t_1)]$, we obtain the fixed-point formula (37).

D.5 Definitions of $M(t_1)$, $\mathcal{M}$, $\mathcal{E}_m$, $A(t_1)$ and $\mathcal{E}_a$

Use $\tilde{M}(t_1)$ from (156), and replace $\mathcal{E}_5$ with $\mathcal{E}_m\equiv\mathcal{E}_5$ that was defined in (147):

$$\mathcal{E}_m=\frac{\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}-\frac{\rho_1}{\theta_1}+\frac{\gamma_1}{(1-\gamma_1)}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{\rho_1}{\theta_1}+\frac{1}{\sigma_1}(t_1-1)\lambda_{d1}\right)}{\frac{1}{\rho_1}+\frac{1}{\theta_1}\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\sigma_1}(1-\lambda_{d1})}>0.$$

Then we define $M(t_1)$ as

$$\begin{aligned}M(t_1) &\equiv \frac{\theta_1}{(\theta_1-\rho_1)^2}\frac{\tilde{M}(t_1)}{(1-\gamma_1)\mathcal{E}_3}\left(1+\frac{\gamma_1}{(1-\gamma_1)}\frac{\rho_1}{\theta_1\sigma_1}(1-\lambda_{d1})\right)\\ &= \frac{\theta_1\left(\mathcal{E}_m-\frac{(t_1-1)}{t_1}\theta_1\right)}{(\theta_1-\rho_1)^2\mathcal{E}_3}\left(1+\frac{\rho_1\gamma_1}{\theta_1\sigma_1(1-\gamma_1)}(1-\lambda_{d1})\right)\frac{\tilde{\lambda}_{d1}}{\lambda_{d1}}\frac{D(t_1)}{A(t_1)}\\ &= \mathcal{M}\times\left(\mathcal{E}_m-\frac{(t_1-1)}{t_1}\theta_1\right)\frac{D(t_1)}{A(t_1)},\end{aligned} \tag{159}$$

where $\mathcal{M}$ is defined by

$$\mathcal{M}\equiv\frac{\theta_1}{(\theta_1-\rho_1)^2\mathcal{E}_3}\left(1+\frac{\rho_1\gamma_1}{\theta_1\sigma_1(1-\gamma_1)}(1-\lambda_{d1})\right)\frac{\tilde{\lambda}_{d1}}{\lambda_{d1}}>0, \tag{160}$$

and the term $A(t_1)$ is given by the denominator of $\tilde{M}(t_1)$ from (156):

$$A(t_1) \equiv \alpha_1 - \tilde{\gamma}_1 + \alpha_2 \left[\left(1 - \tilde{\lambda}_{d1} \right) t_1 \left(1 - \mathcal{E}_4 \right) + \tilde{\lambda}_{d1} \right], \tag{161}$$

and we define $\mathcal{E}_a \equiv [(1 - \tilde{\lambda}_{d1}) t_1 (1 - \mathcal{E}_4) + \tilde{\lambda}_{d1}]$ to obtain expression (39) in the main text.

These expressions give us the definition of $M(t_1)$ used in (38) in the main text. To establish the sign of $\mathcal{E}_a$, we use $\mathcal{E}_4$ from (146) to note that

$$\mathcal{E}_4 \left(1 - \lambda_{d1} \right) = \frac{\frac{1}{\theta_1} \frac{1+(1-\gamma_1)(\sigma_1-1)}{\sigma_1(1-\gamma_1)} \left(1 - \lambda_{d1} \right)}{\frac{1}{\rho_1} + \frac{1}{\theta_1} \frac{\gamma_1}{(1-\gamma_1)} \frac{1}{\sigma_1} \left(1 - \lambda_{d1} \right)} < 1.$$

Therefore, $1 - \mathcal{E}_4 > 1 - \frac{1}{(1-\lambda_{d1})} = -\frac{\lambda_{d1}}{(1-\lambda_{d1})}$, and it follows using (84) that

$$\mathcal{E}_a = (1 - \tilde{\lambda}_{d1}) t_1 \left(1 - \mathcal{E}_4 \right) + \tilde{\lambda}_{d1} > -\frac{t_1 (1 - \tilde{\lambda}_{d1}) \lambda_{d1}}{(1 - \lambda_{d1})} + \tilde{\lambda}_{d1} = 0. \tag{162}$$

D.6 Definitions of $R(t_1)$ and $\mathcal{R}$

The term $R(t_1)$ appearing in (158) is a transformation of $\tilde{R}(t_1)$ from (157):

$$R(t_1) \equiv \frac{\rho_1}{\theta_1 \left(\theta_1 - \rho_1 \right)} \frac{\tilde{R}(t_1)}{(1 - \gamma_1) \mathcal{E}_3} = \frac{\rho_1}{\theta_1 \left(\theta_1 - \rho_1 \right)} \frac{1}{\lambda_{d1}} \frac{\left(\theta_1 - \rho_1 \left(1 - \lambda_{d1} \right) \right) \left(T + \tilde{\gamma}_1 \right) - \theta_1 \rho_1}{\left(1 - \tilde{\gamma}_1 \right) \frac{T}{\rho_1} + (1 - \gamma_1) \tilde{\gamma}_1}, \tag{163}$$

where the equality follows using $T(t_1)$ from (86) in (157). We rewrite this as

$$R(t_1) = \mathcal{R} \times \left[\left(\theta_1 - \rho_1 \left(1 - \lambda_{d1} \right) \right) \left(T(t_1) + \tilde{\gamma}_1 \right) - \theta_1 \rho_1 \right], \mathcal{R} \equiv \frac{\frac{\rho_1}{\theta_1 (\theta_1 - \rho_1) \lambda_{d1}}}{\left(1 - \tilde{\gamma}_1 \right) \frac{T(t_1)}{\rho_1} + (1 - \gamma_1) \tilde{\gamma}_1} > 0. \tag{164}$$

Then expression (40) in the main text follows by using use (87) to rewrite $T(t_1) + \tilde{\gamma}_1 = \frac{1}{\Lambda_1}$, so that $T(t_1)/\rho_1 = (1/\rho_1 \Lambda_1) - \gamma_1$ and

$$\mathcal{R} = \frac{\frac{\rho_1}{\theta_1 (\theta_1 - \rho_1) \lambda_{d1}}}{\left(1 - \tilde{\gamma}_1 \right) \left(\frac{1}{\rho_1 \Lambda_1} - \gamma_1 \right) + (1 - \gamma_1) \tilde{\gamma}_1},$$

which is simplified as expression (41) in the main text.

E Proof of Theorem 1

While a fixed point to (37) exists under general conditions,[37] to establish the properties of this fixed point we rely on a slightly different form of the equation shown by (47) in the main text, and repeated here as

$$H(t_1) \equiv t^{het}\left[1 - \gamma_1 R(t_1)\right] - t_1\left[1 + \alpha_2 M(t_1)\right].$$

As explained in the main text, our approach to proving each part of Theorem 1 is to find high and low tariffs at which the sign of $H(t_1)$ switches, and then we apply the intermediate value theorem to obtain a point where $H(t_1^*) = 0$, which by construction is a fixed-point of (37).

In order to apply the intermediate value theorem, we need to consider values of t_1 below unity, meaning an import subsidy, so the revenue cost of the subsidy needs to be deducted from labor income wL to obtain net income I. With enough roundabout production, it seems possible that at a very low tariff – meaning a very high import subsidy – the revenue-cost of the subsidy could exhaust the labor income of the economy, so that net income $I = wL - B$ is zero. In that case, the there is no consumption by consumers at home (though the labor endowment L is still provided), which is then run like an overseas factory solely for the benefit of foreign consumers. We need to check whether this extreme case can occur, and if it does, we denote that minimal tariff (maximum import subsidy) by $t_1^{\min} \geq 0$. We give a more formal definition with:

Definition 3. $t_1^{\min} \equiv \arg\max_{t_1 \geq 0} \{T(t_1) | T(t_1) = 0\}$*, with* $\lambda_{d1}^{\min}$ *denoting the value of* λ_{d1} *at* $t_1^{\min}$*.*

To explain this definition, notice that using B from the main text in (88) that we can solve for income I as

$$I = wL + B = wL\left[\frac{T(t_1)}{T(t_1) + \alpha_1(T(t_1) - (1 - \tilde{\gamma}_1))}\right]. \tag{165}$$

We see that $I = 0 \iff T(t_1) = 0$. We do not rule out in Definition 3 the possibility that there might be multiple tariffs at which $T(t_1) = 0$, in which case $t_1^{\min}$ is chosen as the maximum of these.[38] To solve for $t_1^{\min}$, we use the market clearing condition (4) together with trade balance

[37] Let $W(t_1)$ denote utility U as a function of the tariff. Provided that $W(t_1)$ is continuous and differentiable, then it will reach some maximum over the (compact) range of all possible tariffs and subsidies, and t_1^* at that maximum will satisfy the first-order condition (37).

[38] If $T(t_1)$ is increasing in t_1 then $t_1^{\min}$ will be unique, and conditions to ensure that are provided in Lemma 7.

(74) to get

$$Y_1 = \alpha_1 I + \tilde{\gamma}_1 \left(\lambda_{d1} Y_1 + \frac{\lambda_{m1}}{t_1} Y_1 \right).$$

If we set $I = 0$ and use $\lambda_{d1} + \lambda_{m1} = 1$, then we solve for

$$1 = \tilde{\gamma}_1 \lambda_{d1}^{\min} + \tilde{\gamma}_1 \frac{1 - \lambda_{d1}^{\min}}{t_1^{\min}} \implies t_1^{\min} = \frac{\tilde{\gamma}_1 (1 - \lambda_{d1}^{\min})}{(1 - \tilde{\gamma}_1 \lambda_{d1}^{\min})}. \tag{166}$$

We see that $t_1^{\min} = 0$ for $\gamma_1 = 0$ because then $\tilde{\gamma}_1 = \gamma_1 \rho_1 = 0$. More generally for $0 \le \gamma_1 \le 1$ we have $0 \le \tilde{\gamma}_1 \le \rho_1$, and it follows from (166) that $0 \le t_1^{\min} \le \tilde{\gamma}_1$. Because we solved for $t_1^{\min}$ from the market clearing condition when $I = 0$, it follows from (165) that $T(t_1^{min}) = 0$. Negative income is not possible, so at higher tariffs we have $I > 0$ and then $T(t_1) > 0$ from (165).

Remark 2. *We henceforth restrict our attention to tariffs in the range $t_1 \ge t_1^{\min}$, where $T^{\min} \equiv T(t_1^{\min}) = 0$ and $T(t_1) > 0 \iff t_1 > t_1^{\min}$.*

Before proceeding with the proof of Theorem 1, we make use of the $T(t_1)$ function to slightly transform the terms used within $D(t_1)$ and $M(t_1)$, as defined in Appendix D.4 and D.5. We first transform the elasticity $\mathcal{E}_3$ appearing in (144) using $T(t_1)$ in (86) to obtain

$$\mathcal{E}_3 = \left(\frac{T(t_1) + \tilde{\gamma}_1}{T(t_1)} + \frac{1}{(1 - \gamma_1)(\sigma_1 - 1)} \right) > 0,$$

and so

$$\mathcal{E}_3 T(t_1) = \left(1 + \frac{1}{(1 - \gamma_1)(\sigma_1 - 1)} \right) T + \tilde{\gamma}_1 = (1 - \tilde{\gamma}_1) \frac{T(t_1)}{(1 - \gamma_1)\rho_1} + \tilde{\gamma}_1.$$

Using the above equation with (15) and (152), and noting that $\frac{1+(1-\gamma_1)(\sigma_1-1)}{(1-\gamma_1)(\sigma_1-1)} = \frac{1-\tilde{\gamma}_1}{(1-\gamma_1)\rho_1}$, we obtain

$$D(t_1)T(t_1) = \frac{1 - \tilde{\gamma}_1}{(1 - \gamma_1)\rho_1} \left[T(t_1) - \frac{\tilde{\sigma}_2 (1 - \gamma_1)\rho_1}{(\tilde{\sigma}_2 - 1)} - \left(T(t_1) + (1 - \gamma_1)\rho_1 \frac{\tilde{\gamma}_1}{1 - \tilde{\gamma}_1} \right) (1 - \lambda_{d1}) \mathcal{E}_4 \right]. \tag{167}$$

Also, note that (161) can be rewritten using $T(t_1)$ from (86) as

$$A(t_1) = \alpha_1 - \tilde{\gamma}_1 + \alpha_2 \left[\left(T(t_1) + \tilde{\gamma}_1 - \tilde{\lambda}_{d1} \right) (1 - \mathcal{E}_4) + \tilde{\lambda}_{d1} \right]. \tag{168}$$

We now prove Theorem 1 by a series of Definitions, Lemmas and Remarks.

From (164) we have $R(t_1) = \mathcal{R} \times [(\theta_1 - \rho_1 (1-\lambda_{d1}))(T(t_1)+\tilde{\gamma}_1) - \theta_1\rho_1]$, where $\mathcal{R} > 0$. It appears that the term $[(\theta_1 - \rho_1 (1-\lambda_{d1}))(T(t_1)+\tilde{\gamma}_1) - \theta_1\rho_1]$ can be zero, particularly as $T(t_1)$ is low, so that $R(t_1) = 0$. For the proof of parts (a) and (c) in Theorem 1, we will make extensive use of this low tariff, which is defined more formally as follows:

Definition 4. *Define* $t^{R0} \equiv \arg\max_{t_1 \geq t_1^{min}} \{R(t_1) | R(t_1) = 0\}$ *and denote* $T^{R0} \equiv T(t^{R0})$, *where it is understood that* T^{R0} *uses the shares* $\tilde{\lambda}_{d1}^{R0}$ *and* λ_{d1}^{R0} *which are evaluated at* t^{R0}.

This definition allows for the possibility that there could be multiple tariffs at which $R(t_1) = 0$, in which case t^{R0} is chosen as the maximum of these points.

Lemma 3. *The tariff* t^{R0} *is given by*

$$t^{R0} = 1 + \frac{\rho_1}{\left(1-\tilde{\lambda}_{d1}^{R0}\right)} \frac{\rho_1\left(1-\lambda_{d1}^{R0}\right)}{\theta_1 - \rho_1\left(1-\lambda_{d1}^{R0}\right)} - \frac{(1-\rho_1)}{\left(1-\tilde{\lambda}_{d1}^{R0}\right)}, \tag{169}$$

with $t_1^{\min} < t^{R0} < 1$ *and* $R(t_1) > 0$ *for* $t_1 > t^{R0}$.

Proof: Because $\mathcal{R} > 0$ in (40), then $R(t_1) = 0$ implies $[(\theta_1 - \rho_1 (1-\lambda_{d1}))(T(t_1)+\tilde{\gamma}_1) - \theta_1\rho_1] = 0$. This condition is rewritten as

$$T^{R0} = \frac{\theta_1\rho_1}{\theta_1 - \rho_1\left(1-\lambda_{d1}^{R0}\right)} - \tilde{\gamma}_1 = \left(\frac{\theta_1}{\theta_1 - \rho_1\left(1-\lambda_{d1}^{R0}\right)} - \gamma_1\right)\rho_1 > 0, \tag{170}$$

because the first ratio on the right is greater than 1 and so it exceeds γ_1. It follows from Remark 2 that $t^{R0} > t_1^{\min}$, and from Definition 4 that $R(t_1) > 0$ for $t_1 > t^{R0}$.

Using (86) we can solve for t^{R0} to obtain obtain (169), which can also be written as

$$t^{R0} = 1 + \frac{1}{\left(1-\tilde{\lambda}_{d1}^{R0}\right)} \left(\frac{-(1-\rho_1)\theta_1 + \rho_1\left(1-\lambda_{d1}^{R0}\right)}{\theta_1 - \rho_1\left(1-\lambda_{d1}^{R0}\right)}\right) < 1$$

where the final inequality follows from $\theta_1 > (\sigma_1 - 1) \Rightarrow \theta_1 (1-\rho_1) > \rho_1 \left(1-\lambda_{d1}^{R0}\right)$. QED

E.1 PROOF OF PART (a)

We assume that $\alpha_1 = 1$, and then $H(t_1)$ from (47) becomes $H(t_1) = t^{het} - t_1 - t^{het}\gamma_1 R(t_1)$. With $R(t^{R0}) = 0$ for $t^{R0} < 1$, we obtain $H(t^{R0}) = t^{het} - t^{R0} > 0$. Checking the sign of $R(t^{het})$, because $T(t^{het}) > 1 - \tilde{\gamma}_1$ it readily follows from (163) that $R(t^{het}) > 0$. In that case we obtain $H(t^{het}) = -t^{het}\gamma_1 R(t^{het}) < 0$ for $\gamma_1 > 0$. Using the continuity of $R(t_1)$ and therefore of $H(t_1)$, it follows from the intermediate value theorem that there exists a tariff t_1^* with $t^{R0} < t_1^* < t^{het}$ at which $H(t_1^*) = 0$. By construction, this tariff is a fixed point of (37). QED

E.2 PROOF OF PARTS (b) AND (c)

From (159) we have $M(t_1) = \mathcal{M} \times \left(\mathcal{E}_m - \frac{(t_1-1)}{t_1}\theta_1\right)\frac{D(t_1)}{A(t_1)}$, where $\mathcal{M} > 0$ from (160). It appears to be possible that $M(t_1) = 0$ for two reasons: either $D(t_1) = 0$ at some tariff; or $\mathcal{E}_m - \frac{(t_1-1)}{t_1}\theta_1$ at some tariff. For the proof of parts (b) and (c) in Theorem 1, we will make extensive use of the first of these points, where $D(t_1) = 0$, which is defined more formally as follows:

Definition 5. *Define*

$$t^{D0} \equiv \begin{cases} \arg\min_{t_1 \geq t_1^{\min}} \{D(t_1) = 0\} & \textit{if this value exists,} \\ +\infty & \textit{otherwise,} \end{cases}$$

and denote $T^{D0} \equiv T(t_1^{D0})$ and likewise for the shares $\tilde{\lambda}_{d1}^{D0}$ and λ_{d1}^{D0} evaluated at t_1^{D0}.

Once again, we allow for multiple solutions for the tariff where $D(t_1) = 0$, and in this case we choose t^{D0} as the minimum of them. Next, we establish a result for the term $\mathcal{E}_m - \frac{(t_1-1)}{t_1}\theta_1$ that also appears within $M(t_1) = \mathcal{M} \times \left(\mathcal{E}_m - \frac{(t_1-1)}{t_1}\theta_1\right)\frac{D(t_1)}{A(t_1)}$, and could possibly make this expression equal to zero.

Lemma 4. $\mathcal{E}_m - \frac{(t_1-1)}{t_1}\theta_1 > 0$ *for all* $t_1 \in [t_1^{\min}, t^{het}]$*. In addition, if* $\gamma_1 = 0$ *then* $\mathcal{E}_m - \frac{(t_1''-1)}{t_1''}\theta_1 = 0$ *at a tariff* $t_1'' > t^{het}$*.*

Proof: From (147) we see that

$$\mathcal{E}_m - \frac{(t_1-1)}{t_1}\theta_1 = \frac{\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}} - \frac{\rho_1}{\theta_1} + \frac{\gamma_1}{(1-\gamma_1)}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{\rho_1}{\theta_1} + \frac{1}{\sigma_1}(t_1-1)\lambda_{d1}\right)}{\frac{1}{\rho_1} + \frac{\gamma_1(1-\lambda_{d1})}{(1-\gamma_1)\sigma_1\theta_1}} - \frac{(t_1-1)}{t_1}\theta_1. \tag{171}$$

Notice that the final term on the right is increasing in t_1, so it takes its highest value over $t_1 \in$

$[t_1^{\min}, t^{het}]$ at $t_1 = t^{het}$, in which that term equals ρ_1. It follows that

$$\begin{aligned}
\mathcal{E}_m &- \frac{(t_1-1)}{t_1}\theta_1 \geq \frac{\frac{1+\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}} - \frac{\rho_1}{\theta_1} + \frac{\gamma_1}{(1-\gamma_1)}\frac{1-\tilde{\lambda}_{d1}}{\tilde{\lambda}_{d1}}\left(\frac{\rho_1}{\theta_1} + \frac{1}{\sigma_1}(t_1-1)\lambda_{d1}\right)}{\frac{1}{\rho_1} + \frac{\gamma_1(1-\lambda_{d1})}{(1-\gamma_1)\sigma_1\theta_1}} - \rho_1 \\
&= \frac{\rho_1}{\tilde{\lambda}_{d1}}\left(\frac{1 - \frac{\rho_1}{\theta_1}\tilde{\lambda}_{d1} + \frac{\gamma_1}{(1-\gamma_1)}\left(1-\tilde{\lambda}_{d1}\right)\frac{\rho_1}{\theta_1} + \frac{\gamma_1}{(1-\gamma_1)\sigma_1}\left((1-\tilde{\lambda}_{d1})(t_1-1)\lambda_{d1} - \frac{\rho_1}{\theta_1}(1-\lambda_{d1})\tilde{\lambda}_{d1}\right)}{1 + \rho_1\frac{\gamma_1(1-\lambda_{d1})}{(1-\gamma_1)\sigma_1\theta_1}}\right) \\
&= \frac{\rho_1}{\tilde{\lambda}_{d1}}\left(\frac{1 - \frac{\rho_1}{\theta_1}\tilde{\lambda}_{d1} + \frac{\gamma_1}{(1-\gamma_1)}\left(1-\tilde{\lambda}_{d1}\right)\frac{\rho_1}{\theta_1} + \frac{\gamma_1}{(1-\gamma_1)\sigma_1}(1-\tilde{\lambda}_{d1})\lambda_{d1}\left(\frac{t_1}{t^{het}} - 1\right)}{1 + \rho_1\frac{\gamma_1(1-\lambda_{d1})}{(1-\gamma_1)\sigma_1\theta_1}}\right),
\end{aligned}$$

where the final line follows using (84). The second two terms in the numerator are

$$\frac{\gamma_1}{(1-\gamma_1)}\left(1-\tilde{\lambda}_{d1}\right)\left(\frac{\rho_1}{\theta_1} - \frac{\lambda_{d1}}{\sigma_1}\left(1 - \frac{t_1}{t^{het}}\right)\right) \geq \frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\theta_1\sigma_1}(1-\tilde{\lambda}_{d1})\left[t_1(\theta_1-\rho_1) - (\theta_1 - (\sigma_1-1))\right],$$

which is positive for $t_1 \geq 1$ and proves that $\mathcal{E}_m - \frac{(t_1-1)}{t_1}\theta_1 > 0$ for all $t_1 \in [t_1^{\min}, t^{het}]$.

To evaluate (171) at higher levels of the tariff, note that with $\gamma_1 = 0$ we have that

$$\lim_{t_1\to\infty}\left(\mathcal{E}_m|_{\gamma_1=0} - \frac{(t_1-1)}{t_1}\theta_1\right) = \rho_1\left(2 - \frac{\rho_1}{\theta_1}\right) - \theta_1 < 0$$

because $\left(2 - \frac{\rho_1}{\theta_1} - \frac{\theta_1}{\rho_1}\right) < 0$ for $\frac{\theta_1}{\rho_1} > 1$. It follows that for $\gamma_1 = 0$ then there exists a tariff $t_1'' > t^{het}$ at which $\mathcal{E}_m - \frac{(t_1''-1)}{t_1''}\theta_1 = 0$. QED

E.3 PROOF OF PART (b)(i)

If $\gamma_s = 0$ for $s = 1, 2$, then from (144) and (146) we have $\mathcal{E}_3 = \frac{\sigma_1}{(\sigma_1-1)}$ and $\mathcal{E}_4 = \frac{\rho_1}{\theta_1}$. Substituting these into (152) we obtain

$$\begin{aligned}
D(t_1) &= \left[\frac{\sigma_1}{(\sigma_1-1)} - \frac{\sigma_2}{(\sigma_2-1)}\frac{1}{T(t_1)} - \frac{1}{\theta_1}(1-\lambda_{d1})\right] \\
&> \left[\frac{\sigma_1}{(\sigma_1-1)} - \frac{\sigma_2}{(\sigma_2-1)} - \frac{1}{\theta_1}\right],
\end{aligned} \tag{172}$$

where the inequality follows from $T(t_1) \geq 1$ for $t_1 \in [1, t^{het}]$ and $\lambda_{d1} < 1$. It follows that $D(t_1) > 0$ when condition (42) holds.

Notice that when $\gamma_1 = 0$ and $\mathcal{E}_4 = \frac{\rho_1}{\theta_1}$ then $A(t_1)$ in (161) becomes

$$A(t_1) \equiv \alpha_1 + \alpha_2 \left[\left(1 - \tilde{\lambda}_{d1}\right) t_1 \left(1 - \frac{\rho_1}{\theta_1}\right) + \tilde{\lambda}_{d1} \right] > \alpha_1 > 0, \tag{173}$$

which is bounded away from zero so that $M(t_1)$ is continuous. Then because $\mathcal{E}_m - \frac{(t_1-1)}{t_1}\theta_1 > 0$ for $t_1 \in [1, t^{het}]$ from Lemma 4, it follows that $M(t_1) > 0$ in that interval, and in particular $M(t^{het}) > 0$. From (47) with $\gamma_1 = 0$ it follows that $H(t^{het}) = -t^{het}\alpha_2 M(t^{het}) < 0$.

Now we make use of t^{D0} which for $\gamma_1 = 0$ is solved by setting (172) equal to zero, giving

$$\frac{\sigma_1}{(\sigma_1 - 1)} - \frac{\sigma_2}{(\sigma_2 - 1)} \frac{1}{T(t^{D0})} - \frac{1}{\theta_1}(1 - \lambda_{d1}) = 0. \tag{174}$$

It follows that

$$T(t^{D0}) = \frac{\frac{\sigma_2}{\sigma_2 - 1}}{\frac{\sigma_1}{(\sigma_1-1)} - \frac{(1-\lambda_{d1})}{\theta_1}} < \frac{\frac{\sigma_2}{\sigma_2 - 1}}{\frac{\sigma_1}{(\sigma_1-1)} - \frac{1}{\theta_1}} < 1,$$

from condition (42) . Because $T(t^{D0}) = 1 + \left(t^{D0} - 1\right)\left(1 - \tilde{\lambda}_{d1}\right)$ when $\gamma_1 = 0$, it follows immediately that $t^{D0} < 1$.

We have already shown $H(t^{het}) = -t^{het}\alpha_2 M(t^{het}) < 0$. Since $t^{D0} < 1$ then $M(t^{D0}) = 0$ and so $H(t^{D0}) = t^{het} - t^{D0}\left[1 + \alpha_2 M(t^{D0})\right] = t^{het} - t^{D0} > 0$. Using the continuity of $M(t_1)$ and therefore of $H(t_1)$, it follows from the intermediate value theorem that there exists a tariff t_1^* with $t_1^{D0} < t_1^* < t^{het}$ at which $H(t_1^*) = 0$. By construction, this tariff is a fixed point of (37). QED

E.4 PROOF OF PART (b)(ii)

We define $T^{het} \equiv T(t^{het})$ and $D^{het} \equiv D(t^{het})$. It follows from substituting the expenditure share (71) into the production share (81) and then into $T(t_1)$ in (86) that

$$T^{het} = 1 - \tilde{\gamma}_1 + \frac{\left(t^{het} - 1\right)(1 - \lambda_{d1})}{1 + (t^{het} - 1)\lambda_{d1}}. \tag{175}$$

From the first line of (172) we have $D(t^{het}) = \left[\frac{\sigma_1}{(\sigma_1-1)} - \frac{\sigma_2}{(\sigma_2-1)}\frac{1}{T^{het}} - \frac{1}{\theta_1}(1 - \lambda_{d1})\right]$. With $\gamma_1 = 0$ and $\lambda_{d1} > 0$ we see that $T^{het} < t^{het}$ so that $D(t^{het}) < \left[\frac{\sigma_1}{(\sigma_1-1)} - \frac{\sigma_2}{(\sigma_2-1)}\frac{1}{t^{het}} - \frac{1}{\theta_1}(1 - \lambda_{d1})\right]$. We therefore have $D(t^{het}) < 0$ if

$$\left[\frac{\sigma_1}{(\sigma_1-1)} - \frac{\sigma_2}{(\sigma_2-1)}\frac{1}{t^{het}} - \frac{1}{\theta_1}(1-\lambda_{d1})\right] \leq 0 \iff \frac{\sigma_2}{(\sigma_2-1)} \geq t^{het}\frac{\sigma_1}{(\sigma_1-1)} - \frac{t^{het}}{\theta_1}(1-\lambda_{d1}).$$

A sufficient condition for this inequality to hold is given in (b)(ii). Using Lemma 4 it follows that $M(t^{het}) < 0$, and therefore from $H(t_1)$ in (47), with $\gamma_1 = 0$ we have $H(t^{het}) = -t^{het}\alpha_2 M(t^{het}) > 0$.

Now we check a higher tariff $t_1'' > t^{het}$ from Lemma 4 at which $\mathcal{E}_m - \frac{(t_1''-1)}{t_1''}\theta_1 = 0$ and therefore $M(t_1'') = 0$. From $H(t_1)$ in (47), with $\gamma_1 = 0$, we have $H(t_1'') = t^{het} - t_1''[1 + \alpha_2 M(t_1'')] = t^{het} - t_1'' < 0$. Using from the continuity of $M(t_1)$ and therefore of $H(t_1)$, it follows from the intermediate value theorem that there exists a tariff t_1^* with $t^{het} < t_1^* < t_1''$ at which $H(t_1^*) = 0$. By construction, this tariff is a fixed point of (37). QED

E.5 PROOF OF PART (c)

We first establish conditions to ensure that $A(t_1) > 0$, starting with the region $t_1 \geq t^{het}$.

Lemma 5. $(1-\mathcal{E}_4)t^{het} \geq \rho_1$ *when condition (43) holds, where $\mathcal{E}_4$ can be evaluated at any tariff. It follows that $A(t_1) > \alpha_1(1-\rho_1) + (1-\gamma_1)\rho_1 > 0$ for all $t_1 \geq t^{het}$.*

Proof: We want to ensure that $(1-\mathcal{E}_4) \geq \frac{\rho_1}{t^{het}} = \frac{\rho_1(\theta_1-\rho_1)}{\theta_1}$. Use (146) to obtain

$$1 - \frac{\frac{1}{\theta_1}\frac{1+(1-\gamma_1)(\sigma_1-1)}{\sigma_1(1-\gamma_1)}}{\frac{1}{\rho_1} + \frac{1}{\theta_1}\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\sigma_1}(1-\lambda_{d1})} \geq \frac{\rho_1(\theta_1-\rho_1)}{\theta_1}.$$

Then we take $\lambda_{d1} = 1$ to get a sufficient condition

$$1 - \frac{\rho_1}{\theta_1}\frac{1+(1-\gamma_1)(\sigma_1-1)}{\sigma_1(1-\gamma_1)} \geq \frac{\rho_1(\theta_1-\rho_1)}{\theta_1} \iff \frac{\sigma_1}{\rho_1}(\theta_1-\rho_1)(1-\rho_1) \geq \frac{\gamma_1}{(1-\gamma_1)}, \tag{176}$$

which is equivalent to (43) in the main text. Now the magnitude of $A(t_1)$ is established from

$$\begin{aligned} A(t_1) &= \alpha_1 - \tilde{\gamma}_1 + \alpha_2\left[\left(1-\tilde{\lambda}_{d1}\right)t_1(1-\mathcal{E}_4) + \tilde{\lambda}_{d1}\right] \geq \alpha_1 - \tilde{\gamma}_1 + \alpha_2\left[\left(1-\tilde{\lambda}_{d1}\right)t^{het}(1-\mathcal{E}_4) + \tilde{\lambda}_{d1}\right] \\ &\geq \alpha_1 - \gamma_1\rho_1 + \alpha_2\left[\left(1-\tilde{\lambda}_{d1}\right)\rho_1 + \tilde{\lambda}_{d1}\right] > \alpha_1(1-\rho_1) + (1-\gamma_1)\rho_1, \end{aligned}$$

where the first inequality follows from $t_1 \geq t^{het}$, the second from $(1-\mathcal{E}_4)t^{het} \geq \rho_1$, and the final inequality from $[(1-\tilde{\lambda}_{d1})\rho_1 + \tilde{\lambda}_{d1}] > \rho_1$. QED

Next, we define the tariff t^{A0} at which $A(t_1)$ becomes zero, if it exists:

Definition 6. *a) Define*

$$t^{A0} \equiv \begin{cases} \arg\max_{t_1 \geq t_1^{\min}} \{A(t_1) = 0\} & \text{if this value exists,} \\ t_1^{\min} & \text{otherwise,} \end{cases}$$

and denote $T^{A0} \equiv T(t^{A0})$ and likewise for the shares $\tilde{\lambda}_{d1}^{A0}$ and λ_{d1}^{A0}, which are evaluated at t^{A0}.

In this definition we are looking for tariffs at which $A(t_1) = 0$, but there will be no such tariffs if $A(t_1) > 0$ for all $t_1 \geq t_1^{\min}$. In that case, $t^{A0} = t_1^{\min} < 1$. On the other hand, if there are multiple tariffs at which $A(t_1) = 0$, then t^{A0} is the maximum of these. From Lemma 5, which relies on condition (43), we know that $t^{A0} < t^{het}$. In Lemma 8 below, we will further show that condition (44) ensures that $t^{A0} < t^{R0}$, and we know that $t^{R0} < 1$ from Lemma 3, so $t^{A0} < 1$.

Remark 6. *The tariff t^{A0} is the import subsidy referred to as t_1' in the statement of Theorem 1(c).*

Lemma 7. *For $t_1 \in (t_1^{\min}, 1)$, $T(t_1)$ is monotonically increasing in t_1 provided that $A(t_1) > 0$.*

Proof: From (85) combined with (86), $T(t_1)$ is given by

$$T = \frac{\tilde{\lambda}_{d1}}{\lambda_{d1}} - \tilde{\gamma}_1, \tag{177}$$

which we differentiate to obtain,

$$dT = \frac{\tilde{\lambda}_{d1}}{\lambda_{d1}} \left(\hat{\tilde{\lambda}}_{d1} - \hat{\lambda}_{d1} \right).$$

Totally differentiate (84), we can show that

$$\hat{\lambda}_{d1} = \frac{(1-\lambda_{d1})}{\left(1-\tilde{\lambda}_{d1}\right)} \left[\hat{\tilde{\lambda}}_{d1} - \left(1-\tilde{\lambda}_{d1}\right) \hat{t}_1 \right]. \tag{178}$$

Then combining with (149), we obtain

$$\hat{\lambda}_{d1} - \hat{\tilde{\lambda}}_{d1} = \frac{\theta_1}{\tilde{\lambda}_{d1}} \left(\tilde{\lambda}_{d1} - \lambda_{d1} \right) \hat{\varphi}_{x1} - (1-\lambda_{d1})\, \hat{t}_1.$$

It follows that,

$$dT = -\frac{\theta_1}{\lambda_{d1}}\left(\tilde{\lambda}_{d1} - \lambda_{d1}\right)\hat{\varphi}_{x1} + \frac{\tilde{\lambda}_{d1}}{\lambda_{d1}}(1-\lambda_{d1})\hat{t}_1. \tag{179}$$

Notice that the coefficient of $\hat{t}_1$ in the final term is positive. We now show that $\hat{\varphi}_{x1}/\hat{t}_1$ is positive, so then because $\tilde{\lambda}_{d1} < \lambda_{d1}$ for $t_1 < 1$, we have established the monotonicity of $T(t_1)$.

Using (145) and (150), we have

$$\hat{t}_1 = \left(\mathcal{E}_m - t_1\mathcal{E}_4\frac{\left(1-\tilde{\lambda}_{d1}\right)\left(\mathcal{E}_m - \frac{(t_1-1)}{t_1}\theta_1\right)}{\left((1-\tilde{\lambda}_{d1})t_1\left(\mathcal{E}_4-1\right) - \tilde{\lambda}_{d1} - \frac{1}{\alpha_2}\left(\alpha_1-\tilde{\gamma}_1\right)\right)}\right)\quad\hat{\varphi}_{x1},$$

and so

$$\hat{\varphi}_{x1} = \frac{\left(\left(1-\tilde{\lambda}_{d1}\right)t_1\left(\mathcal{E}_4-1\right) - \tilde{\lambda}_{d1} - \frac{1}{\alpha_2}\left(\alpha_1-\tilde{\gamma}_1\right)\right)}{\left((1-\tilde{\lambda}_{d1})t_1\left(\mathcal{E}_4-1\right) - \tilde{\lambda}_{d1} - \frac{1}{\alpha_2}\left(\alpha_1-\tilde{\gamma}_1\right)\right)\mathcal{E}_m - \mathcal{E}_4\left(1-\tilde{\lambda}_{d1}\right)\left(t_1\mathcal{E}_m - \left(t_1-1\right)\theta_1\right)}\hat{t}_1.$$

Multiply the numerator and denominator by $-\alpha_2$ and use (161) to obtain

$$\hat{\varphi}_{x1} = \frac{A}{A\mathcal{E}_m + \alpha_2\mathcal{E}_4(1-\tilde{\lambda}_{d1})\left(t_1\mathcal{E}_m - \left(t_1-1\right)\theta_1\right)}\hat{t}_1.$$

Because $\mathcal{E}_4 > 0$ and $\mathcal{E}_m > 0$, then for $t_1 < 1$ we have $\hat{\varphi}_{x1}/\hat{t}_1 > 0$. QED

Lemma 8. *$A(t_1) > 0$ for $t_1 \in [t^{R0}, t^{het}]$ provided that (43) and (44) hold.*

Proof: There are two cases to consider. The first case is where $t^{A0} = t_1^{min}$ so that $A(t_1) > 0$ for all $t_1 > t_1^{min}$. In that case, the lemma holds trivially.

The second case is where $t^{A0} > t_1^{min}$. Then according to (161), $A(t^{A0}) = 0$ at the tariff

$$\alpha_2\left((1-\tilde{\lambda}_{d1})t^{A0}\left(1-\mathcal{E}_4\right) + \tilde{\lambda}_{d1}\right) + \alpha_1 - \tilde{\gamma}_1 = 0,$$

so that,

$$(1-\tilde{\lambda}_{d1})t^{A0} = -\frac{\alpha_1\left(1-\tilde{\lambda}_{d1}\right) + \tilde{\lambda}_{d1} - \tilde{\gamma}_1}{\left(1-\mathcal{E}_4\right)\alpha_2}. \tag{180}$$

Using the definition of $T(t_1)$ in (86), we can rewrite (161) as

$$\alpha_2\left(T^{A0}-(1-\tilde{\lambda}_{d1})t^{A0}\mathcal{E}_4\right)+\alpha_1\left(1-\tilde{\gamma}_1\right)=0 \implies T^{A0}=(1-\tilde{\lambda}_{d1})t^{A0}\mathcal{E}_4-\frac{\alpha_1}{\alpha_2}\left(1-\tilde{\gamma}_1\right).$$

Combining with (180), T^{A0} can be written as

$$\begin{aligned} T^{A0} &= -\frac{1}{\alpha_2}\left(\frac{\mathcal{E}_4\alpha_2\left(\tilde{\lambda}_{d1}-\tilde{\gamma}_1\right)+\alpha_1\left(1-\tilde{\gamma}_1\right)}{(1-\mathcal{E}_4)}\right) \\ &= -\left(\tilde{\lambda}_{d1}-\tilde{\gamma}_1\right)\frac{\mathcal{E}_4}{(1-\mathcal{E}_4)}-\frac{\alpha_1\left(1-\tilde{\gamma}_1\right)}{\alpha_2\left(1-\mathcal{E}_4\right)}. \end{aligned} \tag{181}$$

We know that the tariff t^{R0} at which $R(t^{R0})=0$ occurs at

$$T^{R0}=\frac{\theta_1\rho_1}{\theta_1-\rho_1\left(1-\lambda_{d1}^{R0}\right)}-\tilde{\gamma}_1.$$

Our goal is to show that $T^{A0}\leq T^{R0}$, which will ensure that $T(t_1)$ is invertible in the range $[t^{A0},1]$ using Lemma 7 with $A(t_1)>0$ in that range. The condition $T^{A0}\leq T^{R0}$ holds if

$$-\left(\tilde{\lambda}_{d1}-\tilde{\gamma}_1\right)\frac{\mathcal{E}_4}{(1-\mathcal{E}_4)}-\frac{\alpha_1\left(1-\tilde{\gamma}_1\right)}{\alpha_2\left(1-\mathcal{E}_4\right)}\leq\frac{\theta_1\rho_1}{\theta_1-\rho_1\left(1-\lambda_{d1}^{R0}\right)}-\tilde{\gamma}_1, \quad \text{or,}$$

$$-\tilde{\lambda}_{d1}\frac{\mathcal{E}_4}{(1-\mathcal{E}_4)}-\frac{\alpha_1\left(1-\tilde{\gamma}_1\right)}{(\alpha_2)\left(1-\mathcal{E}_4\right)}\leq\frac{\theta_1\rho_1}{\theta_1-\rho_1\left(1-\lambda_{d1}^{R0}\right)}-\tilde{\gamma}_1\frac{1}{1-\mathcal{E}_4}.$$

Drop the share on the left and we get the sufficient condition

$$\frac{1}{(1-\mathcal{E}_4)}\frac{(\tilde{\gamma}_1-\alpha_1)}{\alpha_2}\leq\frac{\theta_1\rho_1}{\theta_1-\rho_1\left(1-\lambda_{d1}^{R0}\right)}. \tag{182}$$

If $\alpha_1\geq\tilde{\gamma}_1 \iff \alpha_2\leq 1-\tilde{\gamma}_1$, then this restriction is satisfied. However, for $\alpha_1<\tilde{\gamma}_1$, then we need $\mathcal{E}_4$ sufficiently small so that the above condition holds.

From Lemma 5 we know that $(1-\mathcal{E}_4)t^{het}<1$ and it follows that $\mathcal{E}_4<1$. Then the sufficient condition for (182) is

$$(1-\mathcal{E}_4)\geq\frac{(\tilde{\gamma}_1-\alpha_1)}{\alpha_2}\left[\frac{1}{\rho_1}-\frac{\left(1-\lambda_{d1}^{R0}\right)}{\theta_1}\right].$$

If $\alpha_1\geq\tilde{\gamma}_1$ then this condition is once again satisfied, since then the right-hand side is less than or equal to zero, while the left-hand side is positive. For $\alpha_1 < \tilde{\gamma}_1$, we can take $\lambda_{d1}^{R0} = 1$ to get the sufficient condition

$$1 - \mathcal{E}_4 \geq \frac{(\tilde{\gamma}_1 - \alpha_1)}{\alpha_2}\left[\frac{1}{\rho_1} - \frac{1}{\theta_1} + \frac{1}{\theta_1}\right] \geq \frac{(\tilde{\gamma}_1 - \alpha_1)}{\alpha_2}\left[\frac{1}{\rho_1} - \frac{1}{\theta_1} + \frac{\lambda_{d1}^{R0}}{\theta_1}\right].$$

We can substitute for $\mathcal{E}_4$ and the sufficient condition becomes

$$1 - \frac{\frac{1}{\theta_1} + \frac{1}{\theta_1}\frac{\gamma_1}{\sigma_1(1-\gamma_1)}}{\frac{1}{\rho_1} + \frac{1}{\theta_1}\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\sigma_1}(1-\lambda_{d1})} \geq \frac{(\tilde{\gamma}_1 - \alpha_1)}{\alpha_2 \rho_1}.$$

A sufficient condition for this inequality is obtained by taking $\lambda_{d1} = 1$ on the left, so

$$1 - \frac{\frac{1}{\theta_1} + \frac{1}{\theta_1}\frac{\gamma_1}{\sigma_1(1-\gamma_1)}}{\frac{1}{\rho_1}} \geq \frac{(\tilde{\gamma}_1 - \alpha_1)}{\alpha_2 \rho_1} \implies \alpha_1 \geq \frac{-(1-\gamma_1) + \rho_1\left(\frac{1}{\theta_1} + \frac{1}{\theta_1}\frac{\gamma_1}{\sigma_1(1-\gamma_1)}\right)}{\frac{1}{\rho_1} - 1 + \rho_1\left(\frac{1}{\theta_1} + \frac{1}{\theta_1}\frac{\gamma_1}{\sigma_1(1-\gamma_1)}\right)}.$$

Rewriting this as an upper-bound on α_2, and also using $\alpha_2 \leq 1 - \tilde{\gamma}_1$, we therefore obtain (44) as the sufficient condition for $t^{A0} < t^{R0}$, which ensures the $A(t_1) > 0$ for $t_1 \in [t^{R0}, t^{het}]$. QED

Lemma 9. $D(t^{R0}) < 0$. *It follows by also using conditions (43) and (44) together with Lemma 4 that* $M(t^{R0}) < 0$.

Proof: We evaluate $D(t_1)$ from (152) at t^{R0} where we also evaluate the elasticities $\mathcal{E}_3$, and $\mathcal{E}_4$ at t^{R0}. Then $D(t^{R0}) < 0$ if the following expression is negative

$$\begin{aligned} 1 \quad - \quad & \frac{\frac{1+(1-\gamma_2)(\sigma_2-1)}{(1-\gamma_2)(\sigma_2-1)}(1-\gamma_1)(\sigma_1-1)}{\sigma_1(1-\tilde{\lambda}_{d1})(t^{R0}-1) + (1+(1-\gamma_1)(\sigma_1-1))} \\ - \quad & \frac{(1-\gamma_1)(\sigma_1-1)(1-\lambda_{d1})}{1+(1-\gamma_1)(\sigma_1-1)} \frac{\left(\frac{T(t_1)+\tilde{\gamma}_1}{T(t_1)} + \frac{1}{(1-\gamma_1)(\sigma_1-1)}\right)\frac{1}{\theta_1}\frac{1+(1-\gamma_1)(\sigma_1-1)}{\sigma_1(1-\gamma_1)}}{\frac{1}{\rho_1} + \frac{1}{\theta_1}\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\sigma_1}(1-\lambda_{d1})} < 0. \end{aligned}$$

Using $T^{R0} = \frac{\theta_1\rho_1}{\theta_1 - \rho_1\left(1-\lambda_{d1}^{R0}\right)} - \tilde{\gamma}_1$, then we require

$$\begin{aligned} 1 - & \frac{\frac{1+(1-\gamma_2)(\sigma_2-1)}{1-\gamma_2(\sigma_2-1)}(1-\gamma_1)(\sigma_1-1)}{\sigma_1(1-\tilde{\lambda}_{d1})(t^{R0}-1) + (1+(1-\gamma_1)(\sigma_1-1))} \\ - & \frac{(1-\gamma_1)(\sigma_1-1)(1-\lambda_{d1})}{1+(1-\gamma_1)(\sigma_1-1)} \frac{\left(\frac{\theta_1\rho_1}{\theta_1\rho_1 - \tilde{\gamma}_1\left(\theta_1-\rho_1\left(1-\lambda_{d1}^{R0}\right)\right)} + \frac{1}{(1-\gamma_1)(\sigma_1-1)}\right)\frac{1}{\theta_1}\frac{1+(1-\gamma_1)(\sigma_1-1)}{\sigma_1(1-\gamma_1)}}{\frac{1}{\rho_1} + \frac{1}{\theta_1}\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\sigma_1}(1-\lambda_{d1})} < 0. \end{aligned}$$

Given that $\frac{1+(1-\gamma_2)(\sigma_2-1)}{1-\gamma_2(\sigma_2-1)} > 1$ then a sufficient condition is

$$\frac{1}{\sigma_1(1-\tilde{\lambda}_{d1})\left(t^{R0}-1\right)+\left(1+\left(1-\gamma_1\right)\left(\sigma_1-1\right)\right)} + \frac{\left(1-\lambda_{d1}\right)\left(\frac{\theta_1\rho_1}{\theta_1\rho_1-\tilde{\gamma}_1\left(\theta_1-\rho_1\left(1-\lambda_{d1}^{R0}\right)\right)}+\frac{1}{(1-\gamma_1)(\sigma_1-1)}\right)\frac{1}{\theta_1}\frac{1}{\sigma_1(1-\gamma_1)}}{\frac{1}{\rho_1}+\frac{1}{\theta_1}\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\sigma_1}\left(1-\lambda_{d1}\right)} \geq \frac{1}{\left(1-\gamma_1\right)\left(\sigma_1-1\right)}.$$

Using $t^{R0}=1+\frac{\rho_1}{\left(1-\tilde{\lambda}_{d1}^{R0}\right)}\frac{\rho_1\left(1-\lambda_{d1}^{R0}\right)}{\theta_1-\rho_1\left(1-\lambda_{d1}^{R0}\right)}-\frac{(1-\rho_1)}{\left(1-\tilde{\lambda}_{d1}^{R0}\right)}$, we simplify this inequality as

$$\frac{\theta_1-\rho_1\left(1-\lambda_{d1}^{R0}\right)}{\sigma_1\rho_1^2\left(1-\lambda_{d1}^{R0}\right)+\left(1-\gamma_1\right)\left(\sigma_1-1\right)\left(\theta_1-\rho_1\left(1-\lambda_{d1}^{R0}\right)\right)} + \frac{\left(1-\lambda_{d1}\right)\left(\frac{\theta_1\rho_1}{\theta_1\rho_1-\tilde{\gamma}_1\left(\theta_1-\rho_1\left(1-\lambda_{d1}^{R0}\right)\right)}+\frac{1}{(1-\gamma_1)(\sigma_1-1)}\right)\rho_1\frac{(1-\gamma_1)(\sigma_1-1)}{\sigma_1(1-\gamma_1)}}{\rho_1\frac{1}{(1-\gamma_1)}\frac{1}{\sigma_1}\left(1-\lambda_{d1}\right)\left(1-\gamma_1\right)\left(\sigma_1-1\right)+\left(1-\gamma_1\right)\left(\sigma_1-1\right)\left(\theta_1-\rho_1\frac{1}{\sigma_1}\left(1-\lambda_{d1}\right)\right)} \geq \frac{1}{\left(1-\gamma_1\right)\left(\sigma_1-1\right)}.$$

With further simplifications, this inequality is expressed as

$$\begin{aligned} &\frac{\theta_1-\rho_1\left(1-\lambda_{d1}^{R0}\right)}{\sigma_1\rho_1^2\left(1-\lambda_{d1}^{R0}\right)+\left(1-\gamma_1\right)\left(\sigma_1-1\right)\left(\theta_1-\rho_1\left(1-\lambda_{d1}^{R0}\right)\right)} \\ + \quad &\frac{\left(\frac{\theta_1\rho_1(1-\lambda_{d1})(\sigma_1-1)}{(1-\gamma_1)\theta_1+\gamma_1\rho_1\left(1-\lambda_{d1}^{R0}\right)}+-\sigma_1\theta_1+\rho_1\left(1-\lambda_{d1}\right)\right)}{\sigma_1\rho_1^2\left(1-\lambda_{d1}\right)+\left(1-\gamma_1\right)\left(\sigma_1-1\right)\left(\sigma_1\theta_1-\rho_1\left(1-\lambda_{d1}\right)\right)} \geq 0 \iff \\ &\frac{1}{\sigma_1\rho_1^2\left(1-\lambda_{d1}^{R0}\right)+\left(1-\gamma_1\right)\left(\sigma_1-1\right)\left(\theta_1-\rho_1\left(1-\lambda_{d1}^{R0}\right)\right)} \\ \geq \quad &\frac{\frac{(\sigma_1-1)\theta_1(1-\gamma_1)}{(1-\gamma_1)\theta_1+\gamma_1\rho_1\left(1-\lambda_{d1}^{R0}\right)}+1}{\sigma_1\rho_1^2\left(1-\lambda_{d1}\right)+\left(1-\gamma_1\right)\left(\sigma_1-1\right)\left(\sigma_1\theta_1-\rho_1\left(1-\lambda_{d1}\right)\right)}. \end{aligned}$$

Cross-multiplying terms we obtain

$$1+\frac{\left(1-\gamma_1\right)\left(\sigma_1-1\right)\left(\sigma_1-1\right)\theta_1}{\sigma_1\rho_1^2\left(1-\lambda_{d1}^{R0}\right)+\left(1-\gamma_1\right)\left(\sigma_1-1\right)\left(\theta_1-\rho_1\left(1-\lambda_{d1}^{R0}\right)\right)} \geq \frac{\left(\sigma_1-1\right)\theta_1(1-\gamma_1)}{(1-\gamma_1)\theta_1+\gamma_1\rho_1\left(1-\lambda_{d1}^{R0}\right)}+1,$$

so we obtain the inequality $(1-\gamma_1)\theta_1+\gamma_1\rho_1\left(1-\lambda_{d1}^{R0}\right) \geq \rho_1\left(1-\lambda_{d1}^{R0}\right)+(1-\gamma_1)\left(\theta_1-\rho_1\left(1-\lambda_{d1}^{R0}\right)\right)$ which is true because by canceling common terms, it holds as an equality. QED

To prove Theorem 1(c), we rely on two, final Lemmas.

Lemma 10. *If* $D^{het} \equiv D(t^{het}) < 0$, *then provided that condition (45) holds, if follows that*

$$D^{het} > \left[\frac{\tilde{\sigma}_1}{(\tilde{\sigma}_1 - 1)} \frac{(t^{het} - \tilde{\gamma}_1)}{(1 - \tilde{\gamma}_1)} - \frac{\tilde{\sigma}_2}{(\tilde{\sigma}_2 - 1)} - \frac{(t^{het} - \tilde{\gamma}_1)}{(1 - \tilde{\gamma}_1)} \mathcal{E}_d(t^{het})|_{\lambda_{d1}=0} \right], \tag{183}$$

where,

$$\mathcal{E}_d(t^{het})|_{\lambda_{d1}=0} = \left[\frac{\tilde{\sigma}_1}{(\tilde{\sigma}_1 - 1)} + \frac{\gamma_1 (\sigma_1 - 1)}{\tilde{\sigma}_1 + \sigma_1 (t^{het} - 1)} \right] \frac{\rho_1 \tilde{\sigma}_1}{\theta_1 \sigma_1 (1 - \gamma_1) + \tilde{\gamma}_1}. \tag{184}$$

Proof: As before, we define $T^{het} \equiv T(t^{het})$. From (167), $D^{het} T^{het}$ equals

$$\left(\frac{1 - \tilde{\gamma}_1}{(1 - \gamma_1)\rho_1} \right) \left[T^{het} \left(1 - (1 - \lambda_{d1}) \mathcal{E}_4\right) - \frac{\tilde{\sigma}_2 (1 - \gamma_1)\rho_1}{(\tilde{\sigma}_2 - 1)} - \frac{\tilde{\gamma}_1 (1 - \gamma_1)\rho_1}{1 - \tilde{\gamma}_1} (1 - \lambda_{d1}) \mathcal{E}_4 \right]. \tag{185}$$

It should be understood that the shares appearing in these equations are also evaluated at t^{het}. Our strategy, however, is to treat these shares as parameters and differentiate $D^{het} T^{het}$ with respect to the share λ_{d1} so as to obtain a lower-bound on $D^{het} T^{het}$. During this process, we are allowing the production share to adjust parametrically according to (81).

The value $D^{het} T^{het}$ changes with the share λ_{d1} according to

$$\frac{\partial D^{het} T^{het}}{\partial \lambda_{d1}} = \left(\frac{1 - \tilde{\gamma}_1}{(1 - \gamma_1)\rho_1} \right) \left[\frac{\partial T^{het}}{\partial \lambda_{d1}} \left(1 - (1 - \lambda_{d1}) \mathcal{E}_4\right) - \left(T^{het} + \frac{\tilde{\gamma}_1 (1 - \gamma_1)\rho_1}{1 - \tilde{\gamma}_1} \right) \frac{\partial (1 - \lambda_{d1}) \mathcal{E}_4}{\partial \lambda_{d1}} \right]. \tag{186}$$

From (175) we have

$$\frac{\partial T^{het}}{\partial \lambda_{d1}} = -\frac{(t^{het} - 1)}{1 + (t^{het} - 1)\lambda_{d1}} - \frac{(t^{het} - 1)^2 (1 - \lambda_{d1})}{[1 + (t^{het} - 1)\lambda_{d1}]^2} = -\frac{(t^{het} - 1)\, t^{het}}{[1 + (t^{het} - 1)\lambda_{d1}]^2}.$$

Also from (146) we see that

$$(1 - \lambda_{d1}) \mathcal{E}_4 = \frac{\frac{(1 - \lambda_{d1})}{\theta_1} + \frac{1}{\theta_1} \frac{\gamma_1}{\sigma_1 (1 - \gamma_1)} (1 - \lambda_{d1})}{\frac{1}{\rho_1} + \frac{1}{\theta_1} \frac{\gamma_1}{(1 - \gamma_1)} \frac{1}{\sigma_1} (1 - \lambda_{d1})} = \frac{\frac{1}{\theta_1} + \frac{1}{\theta_1} \frac{\gamma_1}{\sigma_1 (1 - \gamma_1)}}{\frac{1}{\rho_1 (1 - \lambda_{d1})} + \frac{1}{\theta_1} \frac{\gamma_1}{(1 - \gamma_1)} \frac{1}{\sigma_1}},$$

and so

$$\frac{\partial (1 - \lambda_{d1}) \mathcal{E}_4}{\partial \lambda_{d1}} = -\frac{\frac{1}{\theta_1} + \frac{1}{\theta_1} \frac{\gamma_1}{\sigma_1 (1 - \gamma_1)}}{\left[\frac{1}{\rho_1 (1 - \lambda_{d1})} + \frac{1}{\theta_1} \frac{\gamma_1}{(1 - \gamma_1)} \frac{1}{\sigma_1} \right]^2} \frac{1}{\rho_1 (1 - \lambda_{d1})^2} = -\mathcal{E}_4 \frac{\frac{1}{\rho_1 (1 - \lambda_{d1})}}{\left[\frac{1}{\rho_1 (1 - \lambda_{d1})} + \frac{1}{\theta_1} \frac{\gamma_1}{(1 - \gamma_1)} \frac{1}{\sigma_1} \right]}.$$

Substituting these expressions into (186), we obtain

$$\begin{aligned}
\frac{\partial D^{het}T^{het}}{\partial \lambda_{d1}} &= -\left(\frac{1-\tilde{\gamma}_1}{(1-\gamma_1)\rho_1}\right)\frac{\left(t^{het}-1\right)t^{het}}{\left[1+(t^{het}-1)\lambda_{d1}\right]^2}\left[1-\left(1-\lambda_{d1}\right)\mathcal{E}_4\right] \\
&+ \left(T^{het}+\frac{\tilde{\gamma}_1(1-\gamma_1)\rho_1}{1-\tilde{\gamma}_1}\right)\mathcal{E}_4\frac{\left(\frac{1-\tilde{\gamma}_1}{(1-\gamma_1)\rho_1}\right)\frac{1}{\rho_1(1-\lambda_{d1})}}{\left[\frac{1}{\rho_1(1-\lambda_{d1})}+\frac{1}{\theta_1}\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\sigma_1}\right]} \\
&= -\left(\frac{1-\tilde{\gamma}_1}{(1-\gamma_1)\rho_1}\right)\frac{\left(t^{het}-1\right)t^{het}}{\left[1+(t^{het}-1)\lambda_{d1}\right]^2}\left[\frac{\frac{1}{\rho_1(1-\lambda_{d1})}-\frac{1}{\theta_1}}{\frac{1}{\rho_1(1-\lambda_{d1})}+\frac{1}{\theta_1}\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\sigma_1}}\right] \\
&+ \left(T^{het}+\frac{\tilde{\gamma}_1(1-\gamma_1)\rho_1}{1-\tilde{\gamma}_1}\right)\mathcal{E}_4\frac{\left(\frac{1-\tilde{\gamma}_1}{(1-\gamma_1)\rho_1}\right)\frac{1}{\rho_1(1-\lambda_{d1})}}{\left[\frac{1}{\rho_1(1-\lambda_{d1})}+\frac{1}{\theta_1}\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\sigma_1}\right]} \\
&> \left\{-\frac{\left(t^{het}-1\right)t^{het}}{\left[1+(t^{het}-1)\lambda_{d1}\right]^2}+\left(T^{het}+\frac{\tilde{\gamma}_1(1-\gamma_1)\rho_1}{1-\tilde{\gamma}_1}\right)\mathcal{E}_4\right\}\frac{\left(\frac{1-\tilde{\gamma}_1}{(1-\gamma_1)\rho_1}\right)\frac{1}{\rho_1(1-\lambda_{d1})}}{\left[\frac{1}{\rho_1(1-\lambda_{d1})}+\frac{1}{\theta_1}\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\sigma_1}\right]}.
\end{aligned}$$

Using (175) it follows that $\frac{\partial D^{het}T^{het}}{\partial \lambda_{d1}} > 0$ if

$$\mathcal{E}_4\frac{1+(1-\gamma_1)\rho_1\tilde{\gamma}_1}{1-\tilde{\gamma}_1} > \frac{\left(t^{het}-1\right)}{1+(t^{het}-1)\lambda_{d1}}\left(\frac{t^{het}}{\left[1+(t^{het}-1)\lambda_{d1}\right]}-\mathcal{E}_4\left(1-\lambda_{d1}\right)\right). \tag{187}$$

Substituting $t^{het}=\frac{\theta_1}{\theta_1-\rho_1}$ so that $t^{het}-1=\frac{\rho_1}{\theta_1-\rho_1}$, we simplify this expression to obtain

$$\left(\mathcal{E}_4-\frac{\rho_1}{\theta_1-\rho_1(1-\lambda_{d1})}\right)\left(\frac{\rho_1(1-\lambda_{d1})}{\theta_1-\rho_1(1-\lambda_{d1})}+1\right) > \mathcal{E}_4\left(1-\frac{1+(1-\gamma_1)\rho_1\tilde{\gamma}_1}{1-\tilde{\gamma}_1}\right).$$

Then we substitute (146) on the left and we use the bound from Lemma 5 on the right, which implies that $\mathcal{E}_4 \leq \frac{\theta_1-\rho_1(\theta_1-\rho_1)}{\theta_1}$, to obtain the sufficient condition

$$\left(\frac{\frac{1}{\theta_1}+\frac{1}{\theta_1}\frac{\gamma_1}{\sigma_1(1-\gamma_1)}}{\frac{1}{\rho_1}+\frac{1}{\theta_1}\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\sigma_1}\left(1-\lambda_{d1}\right)}-\frac{\rho_1}{\theta_1-\rho_1(1-\lambda_{d1})}\right)\left(\frac{\rho_1(1-\lambda_{d1})}{\theta_1-\rho_1(1-\lambda_{d1})}+1\right)$$

$$> \frac{\theta_1-\rho_1\left(\theta_1-\rho_1\right)}{\theta_1}\left(1-\frac{1+(1-\gamma_1)\rho_1\tilde{\gamma}_1}{1-\tilde{\gamma}_1}\right).$$

We set $\lambda_{d1}=0$ on the left to obtain a further sufficient condition

$$\frac{\frac{1}{\theta_1}+\frac{1}{\theta_1}\frac{\gamma_1}{\sigma_1(1-\gamma_1)}}{\frac{1}{\rho_1}+\frac{1}{\theta_1}\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\sigma_1}}-\frac{\rho_1}{\theta_1-\rho_1} > \frac{\theta_1-\rho_1\left(\theta_1-\rho_1\right)}{\theta_1}\left(1-\frac{1+(1-\gamma_1)\rho_1\tilde{\gamma}_1}{1-\tilde{\gamma}_1}\right).$$

After extensive simplification, this inequality is written as

$$\gamma_1^2\rho_1\left(\theta_1-\rho_1\left(\theta_1-\rho_1\right)\right)\left(1+(1-\gamma_1)\rho_1\right)\frac{\theta_1-\rho_1}{\theta_1}+\gamma_1\left(\theta_1-2\rho_1\right)\left(1-\gamma_1\rho_1\right)$$

$$>\sigma_1(1-\gamma_1)\rho_1\left(1-\gamma_1\rho_1\right)-\sigma_1(1-\gamma_1)\gamma_1\left(\theta_1-\rho_1\left(\theta_1-\rho_1\right)\right)\left(1+(1-\gamma_1)\rho_1\right)\left(\theta_1-\rho_1\right).$$

This inequality fails to hold at $\gamma_1=0$, so higher values of γ_1 are needed. The first set of terms on the left will involve a cubic in γ_1, so to avoid that a sufficient condition is obtained by ignoring those (positive) terms, resulting in

$$\begin{aligned}&\gamma_1\left(\theta_1-2\rho_1\right)\left(1-\gamma_1\rho_1\right)-\sigma_1(1-\gamma_1)\rho_1\left(1-\gamma_1\rho_1\right)\\ \geq\;&-\sigma_1(1-\gamma_1)\gamma_1\left(\theta_1-\rho_1\left(\theta_1-\rho_1\right)\right)\left(1+(1-\gamma_1)\rho_1\right)\left(\theta_1-\rho_1\right).\end{aligned}$$

A further simplification is obtained by observing that $(1+(1-\gamma_1)\rho_1)$ on the right is made smaller by replacing it with $(1-\gamma_1\rho_1)$, and dividing out that common term to obtain

$$\gamma_1\left(\theta_1-2\rho_1\right)\geq\sigma_1(1-\gamma_1)\left[\rho_1-\gamma_1\left(\theta_1-\rho_1\left(\theta_1-\rho_1\right)\right)\left(\theta_1-\rho_1\right)\right].$$

A sufficient condition for this inequality to hold is provided by (45).

It follows that we can take $\lambda_{d1}=0$ to obtain a lower-bound for $D^{het}T^{het}$, and also $\tilde{\lambda}_{d1}=0$ from (81). So we set both these shares at zero in (175) and (185) to obtain $T^{het}|_{\lambda_{d1}=0}=t^{het}-\tilde{\gamma}_1$. We go back to the original form of $D(t_1)$ in (35) and note that when evaluating Λ_1 at t^{het} and $\lambda_{d1}=0$ we obtain $\Lambda_1=1/t^{het}$. It follows that

$$D(t^{het})|_{\lambda_{d1}=0}\;T^{het}|_{\lambda_{d1}=0}=\left[\frac{\tilde{\sigma}_1}{(\tilde{\sigma}_1-1)}-\frac{\tilde{\sigma}_2}{(\tilde{\sigma}_2-1)}\frac{(1-\tilde{\gamma}_1)}{(t^{het}-\tilde{\gamma}_1)}-\mathcal{E}_d(t^{het})|_{\lambda_{d1}=0}\right]\left(t^{het}-\tilde{\gamma}_1\right).$$

We use this expression and (175) to obtain a bound on D^{het} of

$$\begin{aligned}D^{het}&=\frac{D^{het}T^{het}}{T^{het}}\geq\frac{D^{het}|_{\lambda_{d1}=0}T^{het}|_{\lambda_{d1}=0}}{T^{het}}\\&=\left[\frac{\tilde{\sigma}_1}{(\tilde{\sigma}_1-1)}\frac{(t^{het}-\tilde{\gamma}_1)}{(1-\tilde{\gamma}_1)}-\frac{\tilde{\sigma}_2}{(\tilde{\sigma}_2-1)}-\frac{(t^{het}-\tilde{\gamma}_1)}{(1-\tilde{\gamma}_1)}\mathcal{E}_d(t^{het})|_{\lambda_{d1}=0}\right]\left(\frac{1-\tilde{\gamma}_1}{1-\tilde{\gamma}_1+\frac{(t^{het}-1)(1-\lambda_{d1})}{1+(t^{het}-1)\lambda_{d1}}}\right).\end{aligned}$$

If $D^{het} < 0$ then the expression in brackets must also be negative. It is multiplied by a term less than unity, so that term is lowered by instead multiplying by unity. It follows that (188) holds.

Using (153) we have $\mathcal{E}_d(t^{het})|_{\lambda_{d1}=0} = \mathcal{E}_3(t^{het})|_{\lambda_{d1}=0}\, \mathcal{E}_4(t^{het})|_{\lambda_{d1}=0}$. These expressions are obtained from (144) and (146), using $\lambda_{d1} = 0 \implies \tilde{\lambda}_{d1} = 0$:

$$\begin{aligned}
\mathcal{E}_3(t^{het})|_{\lambda_{d1}=0} &= \frac{\tilde{\sigma}_1}{(\tilde{\sigma}_1 - 1)} + \frac{\gamma_1 (\sigma_1 - 1)}{\tilde{\sigma}_1 + \sigma_1 (t^{het} - 1)}, \\
\mathcal{E}_4(t^{het})|_{\lambda_{d1}=0} &= \frac{\frac{1}{\theta_1}\frac{1+(1-\gamma_1)(\sigma_1-1)}{\sigma_1(1-\gamma_1)}}{\frac{1}{\rho_1} + \frac{1}{\theta_1}\frac{\gamma_1}{(1-\gamma_1)}\frac{1}{\sigma_1}} = \frac{\rho_1 \tilde{\sigma}_1}{\theta_1 \sigma_1 (1-\gamma_1) + \tilde{\gamma}_1}.
\end{aligned}$$

Multiplying the above two expressions we obtain (189). QED

Lemma 11. *When conditions (43) and (45) hold, then $H^{het} < 0$ for all parameters satisfying (36) when κ_1 is chosen as stated in part (c) of Theorem 1 and κ_0 is specified below.*

$$D^{het} > \left[\frac{\tilde{\sigma}_1}{(\tilde{\sigma}_1 - 1)} \frac{(t^{het} - \tilde{\gamma}_1)}{(1 - \tilde{\gamma}_1)} - \frac{\tilde{\sigma}_2}{(\tilde{\sigma}_2 - 1)} - \frac{(t^{het} - \tilde{\gamma}_1)}{(1 - \tilde{\gamma}_1)} \mathcal{E}_d(t^{het})|_{\lambda_{d1}=0} \right], \tag{188}$$

where,

$$\mathcal{E}_d(t^{het})|_{\lambda_{d1}=0} = \left[\frac{\tilde{\sigma}_1}{(\tilde{\sigma}_1 - 1)} + \frac{\gamma_1 (\sigma_1 - 1)}{\tilde{\sigma}_1 + \sigma_1 (t^{het} - 1)} \right] \frac{\rho_1 \tilde{\sigma}_1}{\theta_1 \sigma_1 (1 - \gamma_1) + \tilde{\gamma}_1}. \tag{189}$$

Proof: Using (47), the needed condition is that

$$-\alpha_2 \mathcal{M}^{het} \left(\mathcal{E}_m^{het} - \frac{(t^{het} - 1)}{t^{het}} \theta_1 \right) \frac{D^{het}}{A^{het}} < \gamma_1 R^{het},$$

where $\mathcal{M}^{het}$, $\mathcal{E}_m^{het}$, D^{het}, A^{het} and R^{het} are all evaluated at t^{het}. The expression in parentheses is positive from Lemma 4, so we can rewrite this expression as

$$D^{het} > \frac{-\gamma_1 R^{het} A^{het}}{\alpha_2 \mathcal{M}^{het} \left(\mathcal{E}_m^{het} - \frac{(t^{het}-1)}{t^{het}} \theta_1 \right)}.$$

If $D^{het} \geq 0$ then this condition is automatically satisfied because the right-hand side is negative by earlier results and our assumption that $\gamma_1 > 0$ from (45). So suppose that $D^{het} < 0$. Using Lemma

10, a sufficient condition for the above inequality to hold is

$$\frac{\tilde{\sigma}_2}{(\tilde{\sigma}_2-1)} < \frac{\tilde{\sigma}_1}{(\tilde{\sigma}_1-1)}\frac{(t^{het}-\tilde{\gamma}_1)}{(1-\tilde{\gamma}_1)} - \frac{(t^{het}-\tilde{\gamma}_1)}{(1-\tilde{\gamma}_1)}\mathcal{E}_d(t^{het})|_{\lambda_{d1}=0} + \frac{\gamma_1 R^{het} A^{het}}{\alpha_2 \mathcal{M}^{het}\left(\mathcal{E}_m^{het} - \frac{(t^{het}-1)}{t^{het}}\theta_1\right)}.$$

Therefore, we satisfy condition (36) by specifying κ_1 as in Theorem 1(c) and a preliminary value for κ_0 of

$$\kappa_0 \equiv \frac{\gamma_1 R^{het} A^{het}}{\alpha_2 \mathcal{M}^{het}\left(\mathcal{E}_m^{het} - \frac{(t^{het}-1)}{t^{het}}\theta_1\right)} - \frac{(t^{het}-\tilde{\gamma}_1)}{(1-\tilde{\gamma}_1)}\mathcal{E}_d(t^{het})|_{\lambda_{d1}=0}. \tag{190}$$

Because many of the variables on the right-hand side of this equation depend on expenditure or production shares, we now develop a lower-bound for κ_0 that is independent of these shares and which can be used for κ_0 in (46).

Using the method in the proof of Lemma 4, we first obtain

$$\mathcal{E}_m^{het} - \frac{(t^{het}-1)}{t^{het}}\theta_1 = \frac{\rho_1}{\tilde{\lambda}_{d1}}\left(\frac{1-\frac{\rho_1}{\theta_1}\tilde{\lambda}_{d1} + \frac{\gamma_1}{(1-\gamma_1)}\frac{\rho_1}{\theta_1}(1-\tilde{\lambda}_{d1})}{1+\rho_1\frac{\gamma_1(1-\lambda_{d1})}{(1-\gamma_1)\sigma_1\theta_1}}\right).$$

We substitute this along with the lower-bound for $A(t^{het})$ from Lemma 5, which we rewrite as $A(t^{het}) > A_1 \equiv \alpha_1(1-\rho_1)+(1-\gamma_1)\rho_1$, together with the expressions for $\mathcal{M}^{het}$ and R^{het}, to obtain the following expression where the expenditure and production shares are evaluated at t^{het}:

$$\begin{aligned}
\kappa_0 &> \frac{\gamma_1 R^{het} A_1}{\alpha_2 \mathcal{M}^{het}\left(\frac{\rho_1}{\tilde{\lambda}_{d1}}\left(\frac{1-\frac{\rho_1}{\theta_1}\tilde{\lambda}_{d1}+\frac{\gamma_1}{(1-\gamma_1)}\frac{\rho_1}{\theta_1}(1-\tilde{\lambda}_{d1})}{1+\rho_1\frac{\gamma_1(1-\lambda_{d1})}{(1-\gamma_1)\sigma_1\theta_1}}\right)\right)} - \frac{(t^{het}-\tilde{\gamma}_1)}{(1-\tilde{\gamma}_1)}\mathcal{E}_d(t^{het})|_{\lambda_{d1}=0} \\
&= \frac{\mathcal{E}_3\lambda_{d1}\gamma_1 R^{het} A_1}{\alpha_2\frac{\rho_1\theta_1}{(\theta_1-\rho_1)^2}\left(1-\frac{\rho_1}{\theta_1}\tilde{\lambda}_{d1}+\frac{\gamma_1}{(1-\gamma_1)}\frac{\rho_1}{\theta_1}(1-\tilde{\lambda}_{d1})\right)} - \frac{(t^{het}-\tilde{\gamma}_1)}{(1-\tilde{\gamma}_1)}\mathcal{E}_d(t^{het})|_{\lambda_{d1}=0} \\
&= \frac{(\theta_1-\rho_1)\,\gamma_1\lambda_{d1}\tilde{R}(t^{het})A_1}{\alpha_2\theta_1^2\left(1-\gamma_1-\frac{\rho_1}{\theta_1}\tilde{\lambda}_{d1}+\gamma_1\frac{\rho_1}{\theta_1}\right)} - \frac{(t^{het}-\tilde{\gamma}_1)}{(1-\tilde{\gamma}_1)}\mathcal{E}_d(t^{het})|_{\lambda_{d1}=0},
\end{aligned} \tag{191}$$

where the second line substitutes for $\mathcal{M}$ from (160) and the third line substitutes for R^{het} from (163) and $\mathcal{E}_3$ from (127).

From (157) we evaluate $\tilde{R}(t^{het})$ using $t^{het} - 1 = \frac{\rho_1}{\theta_1 - \rho_1}$ to obtain

$$\begin{aligned}
\lambda_{d1}\tilde{R}(t^{het}) &= \frac{\theta_1(1-\rho_1) - \rho_1\left(1-\lambda_{d1}\right) + \rho_1(1-\tilde{\lambda}_{d1})\left(\frac{\theta_1 - \rho_1(1-\lambda_{d1})}{\theta_1-\rho_1}\right)}{(1-\gamma_1)\rho_1 + \frac{\rho_1(1-\tilde{\lambda}_{d1})}{\theta_1-\rho_1}} \\
&> \frac{\theta_1(1-\rho_1) - \rho_1\left(1-\lambda_{d1}\right) + \rho_1(1-\tilde{\lambda}_{d1})}{(1-\gamma_1)\rho_1 + \frac{\rho_1(1-\tilde{\lambda}_{d1})}{\theta_1-\rho_1}} = \frac{\theta_1(1-\rho_1) - \rho_1\left(\tilde{\lambda}_{d1} - \lambda_{d1}\right)}{(1-\gamma_1)\rho_1 + \frac{\rho_1(1-\tilde{\lambda}_{d1})}{\theta_1-\rho_1}}.
\end{aligned}$$

To establish a lower-bound that is independent of shares, we evaluate (85) at t^{het} to obtain

$$\tilde{\lambda}_{d1} - \lambda_{d1} = \frac{\rho_1}{(\theta_1-\rho_1)}\lambda_{d1}\left(1-\tilde{\lambda}_{d1}\right) < \frac{\rho_1}{(\theta_1-\rho_1)}\lambda_{d1}\left(1-\lambda_{d1}\right) \leq \frac{\rho_1}{4(\theta_1-\rho_1)}.$$

where the first inequality is obtained because $\tilde{\lambda}_{d1} > \lambda_{d1}$ at $t^{het} > 1$ and the second because the maximum value of $\lambda_{d1}\left(1-\lambda_{d1}\right)$ for $\lambda_{d1} \in [0,1]$ is 0.25. We substitute this above and also specify $\tilde{\lambda}_{d1} = 0$ in the denominator to get the lower-bound:

$$\lambda_{d1}\tilde{R}(t^{het}) > \frac{\theta_1(1-\rho_1) - \frac{\rho_1^2}{4(\theta_1-\rho_1)}}{(1-\gamma_1)\rho_1 + \frac{\rho_1}{\theta_1-\rho_1}}.$$

Substituting this inequality into (191) we find that

$$\kappa_0 > \frac{\gamma_1\left(\theta_1(\theta_1-\rho_1)(1-\rho_1) - \frac{\rho_1^2}{4}\right)A_1}{\alpha_2\left(\theta_1(1-\gamma_1) - \rho_1\tilde{\lambda}_{d1} + \gamma_1\rho_1\right)\left(\rho_1\theta_1(1-\gamma_1) + \frac{\rho_1\theta_1}{\theta_1-\rho_1}\right)} - \frac{(t^{het}-\tilde{\gamma}_1)}{(1-\tilde{\gamma}_1)}\mathcal{E}_d(t^{het})|_{\lambda_{d1}=0}. \quad (192)$$

We specify $\tilde{\lambda}_{d1} = 0$ in the denominator to get the final lower-bound:

$$\kappa_0 > \frac{\gamma_1\left(\theta_1(\theta_1-\rho_1)(1-\rho_1) - \frac{\rho_1^2}{4}\right)\left[\alpha_1(1-\rho_1) + (1-\gamma_1)\rho_1\right]}{\alpha_2\left(\theta_1(1-\gamma_1) + \gamma_1\rho_1\right)\left(\rho_1\theta_1(1-\gamma_1) + \frac{\rho_1\theta_1}{\theta_1-\rho_1}\right)} - \frac{(t^{het}-\tilde{\gamma}_1)}{(1-\tilde{\gamma}_1)}\mathcal{E}_d(t^{het})|_{\lambda_{d1}=0}.$$

In the statement of Theorem 1(c), we use the lower-bound on the right to specify κ_0, which gives a sufficient condition to ensure that $H(t^{het}) < 0$. QED

To complete the proof of part (c) we need to establish the tariff $t_1^* \in (t^{R0}, t^{het})$ with $H(t_1^*) = 0$. Using $R(t^{R0}) = 0$, it follows from (47) that $H(t^{R0}) = \left(t^{het} - t^{R0}\right) - t^{R0}\alpha_2 M(t^{R0}) > 0$, because

$M(t^{R0}) < 0$ from Lemma 9 since $D\left(t^{R0}\right) < 0$. From Lemma 11 we have $H(t^{het}) < 0$. It follows from the continuity of $H(t_1)$ that there exists a tariff $t_1^* < t^{het}$ at which $H(t_1^*) = 0$. QED

E.6 Limiting one-sector $(\alpha_1 = 1)$ economy as $\lambda_{d1} \to 0$

Consider the one-sector economy, $\alpha_1 = 1$, where we take the limit $\lambda_{d1} \to 0$. Note from (37) that in the one-sector economy where $\alpha_1 = 1$, the optimal tariff solves

$$t_1^* = t^{het}\left[1 - \gamma_1 R(t_1^*)\right].$$

Using (86) and (164), we find that the optimal tariff t_1^* is implicitly defined by

$$t_1^*\lambda_{d1} = t^{het}\left\{\lambda_{d1} - \gamma_1 \frac{\rho_1}{\theta_1\left(\theta_1 - \rho_1\right)}\left[\frac{\left(\theta_1 - \rho_1(1-\lambda_{d1})\right)\left(1 + \left(t_1^* - 1\right)\left(1 - \tilde{\lambda}_{d1}\right)\right) - \theta_1\rho_1}{\left(1 - \tilde{\gamma}_1\right)\frac{1 - \tilde{\gamma}_1 + \left(t_1^* - 1\right)\left(1 - \tilde{\lambda}_{d1}\right)}{\rho_1} + (1 - \gamma_1)\tilde{\gamma}_1}\right]\right\}.$$

In the limit as $\lambda_{d1} \to 0$, it follows that t_1^* solves

$$0 = \frac{t^{het}\gamma_1\rho_1}{\theta_1\left(\theta_1 - \rho_1\right)}\left[\frac{\left(\theta_1 - \rho_1\right)t_1^* - \theta_1\rho_1}{\left(1 - \tilde{\gamma}_1\right)\frac{t_1^* - \tilde{\gamma}_1}{\rho_1} + (1 - \gamma_1)\tilde{\gamma}_1}\right].$$

Provided that $\gamma_1 > 0$, then this condition is satisfied if and only if the numerator of the term in brackets equals zero, which implies that $t_1^* = \frac{\theta_1\rho_1}{(\theta_1 - \rho_1)}$.

F Equilibrium conditions of the model in relative changes

To calculate the effects of tariffs changes in the quantitative model, we express the equilibrium conditions in relative terms using the "exact-hat" notation for the ratio of after-versus-before levels for a given perturbation, that is, $\widehat{z} = z'/z$ for any variable z. **So the hat notation in this Appendix F differs from the rest of the paper, where $\widehat{z} = d\ln z$.** The equilibrium conditions in Definition 1 can be re-expressed as follows.[39] The change in the price of the input bundle is

$$\hat{c}_s \equiv (\hat{w})^{1-\gamma_s}\left(\hat{P}_s\right)^{\gamma_s},$$

[39]See Appendix C of CFRT (2020) for the derivation of the equations below in a more general model.

and the change in the price index in each sector 1 is

$$\hat{P}_1 = \left(\lambda_{d1}\hat{c}_1^{-\theta_1}\hat{A}_{d1} + \lambda_{m1}\hat{t}_1^{-\theta_1}\hat{A}_{m1}\right)^{-\frac{1}{\theta_1}},$$

where $\lambda_{m1} = (1-\lambda_{d1})$ and $\hat{A}_{m1} \equiv \left(\hat{t}_1/\hat{Y}_1\right)^{\frac{\sigma_1-1-\theta_1}{\sigma_1-1}}$, while in sector 2 we have $\hat{P}_2 = \hat{c}_2\hat{A}_{d2}^{-\frac{1}{\theta_2}}$ with $\hat{A}_{ds} \equiv \hat{N}_s\left(\hat{w}/\hat{Y}_s\right)^{\frac{\sigma_s-1-\theta_s}{\sigma_s-1}}$. The change in the sector 1 domestic share and the export share in the foreign market are

$$\hat{\lambda}_{d1} = \left(\frac{\hat{c}_1}{\hat{P}_1}\right)^{-\theta_s}\hat{A}_{d1} \quad \text{and} \quad \hat{\lambda}_{x1} = \hat{c}_1^{-\theta_1}\hat{A}_{x1},$$

where $\hat{A}_{x1} \equiv \hat{N}_1\hat{w}^{\frac{\sigma_s-1-\theta_s}{\sigma_s-1}}$. The market clearing conditions are,

$$\begin{aligned} Y_1' &= \alpha_1\left(w'L + B'\right) + \tilde{\gamma}_1\left(\lambda_{d1}'Y_1' + \lambda_{x1}'Y_1^*\right), \\ Y_2' &= \alpha_2\left(w'L + B'\right) + \tilde{\gamma}_2 Y_2'. \end{aligned}$$

Tariff revenue in the $'$ equilibrium is given by

$$B' = \frac{(t_1'-1)}{t_1'}\lambda_{m1}'Y_1',$$

and trade balance is

$$\lambda_{x1}'Y_1^* = \frac{\lambda_{m1}'}{t_1'}Y_1',$$

and the final condition for firm entry is

$$\hat{N}_1 = \left(\frac{\lambda_{d1}'Y_1' + \lambda_{x1}'Y_1^*}{\lambda_{d1}Y_1 + \lambda_{x1}Y_1^*}\right)\frac{1}{\hat{w}} \quad \text{and} \quad \hat{N}_2 = \frac{\hat{Y}_2}{\hat{w}}.$$

As we can see, by expressing the model in this way we can analyze the effects of tariff changes without needing information of fixed entry and operating costs. We assume that these fixed costs do not change, and in addition, we have used above the SOE assumption that w^*, Y_1^*, N_1^* and P_1^* are all fixed. The above system of equations can then be used to study the impact of a change in tariffs $\hat{t}_1$ on all equilibrium values, and therefore on indirect utility from (118).

G Quantitative second-best optimal tariffs and parameter values

As explained in the main text, we take two approaches to solve for the optimal tariff. First, we use the system of equations in Appendix F to determine the change in utility for the SOE using the "hat-algebra" method. For each country relative to the rest of the world, we specified a fine grid over the choice of the tariff and numerically computed the change in utility. We chose the tariff t_1^{num} that gave the maximum rise in utility from this numerical approach, and we use the corresponding domestic share λ_{d1}^{num}. Second, we substitute this share into the H function in (47), and then we exactly solve for the optimal tariff t_1^{exact} at which $H(t_1^{exact}) = 0$. It would be possible to iterate on these approaches by using the hat-algebra to further compute λ_{d1}^{exact} for the tariff t_1^{exact}, and then get a more-exact solution for the optimal tariff from $H = 0$. But since the numerical and exact solutions t_1^{num} and t_1^{exact} are very close in nearly all cases, as shown in Figure 5, we judged that further iterations were not needed.

These values of the tariff solutions are shown in Table 3, along with the parameter values for all countries and the domestic share λ_{d1}^{*} and η_{m1}^{*} from (25), both evaluated at t_1^{exact} .

Figure 5: Scatter plot of exact solution versus numerical solution

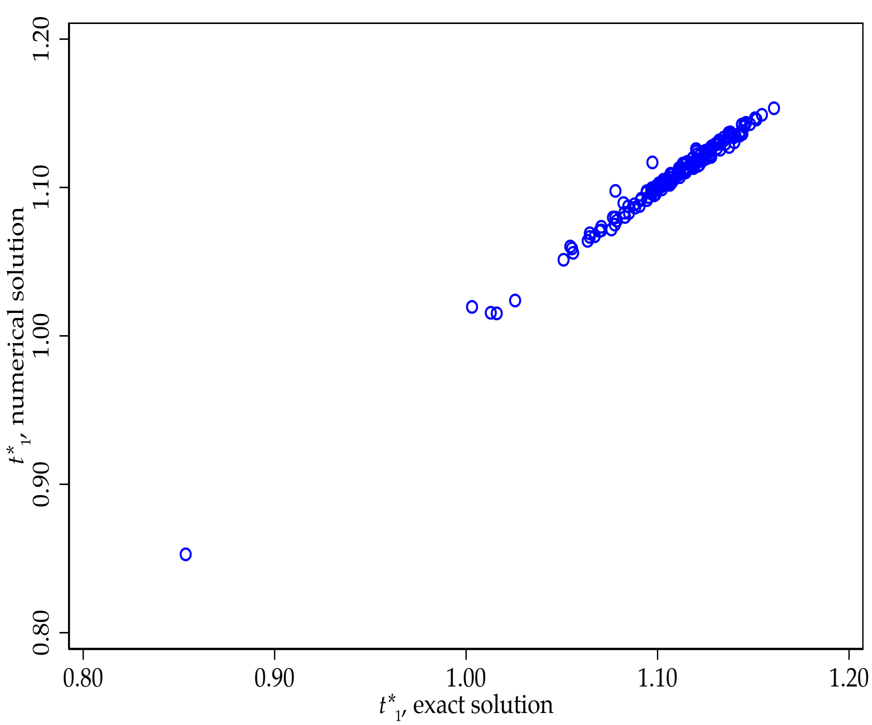

Table 3: Optimal Tariffs and parameters

Country	t_1^* **exact**	t_1^* **numeric**	t^{het}	γ_1	γ_2	α_1	λ_{d1}^*	η_{d1}^*
ABW	1.120	1.126	1.15	0.49	0.11	0.21	0.340	1.54
AGO	1.138	1.137	1.15	0.44	0.13	0.22	0.822	0.14
ALB	1.111	1.113	1.15	0.56	0.17	0.23	0.623	0.55
AND	1.098	1.095	1.15	0.58	0.14	0.21	0.658	0.53
ARE	1.133	1.125	1.15	0.46	0.15	0.25	0.580	0.46
ARG	1.115	1.112	1.15	0.54	0.15	0.28	0.725	0.38
ARM	1.106	1.107	1.15	0.55	0.12	0.24	0.691	0.42
ATG	1.079	1.078	1.15	0.63	0.16	0.24	0.615	1.13
AUS	1.144	1.138	1.15	0.39	0.11	0.19	0.890	0.05
AUT	1.121	1.122	1.15	0.49	0.12	0.27	0.335	1.22
BDI	1.099	1.101	1.15	0.61	0.22	0.26	0.839	0.28
BEL	1.120	1.124	1.15	0.48	0.10	0.27	0.329	1.22
BEN	1.111	1.109	1.15	0.56	0.16	0.22	0.800	0.34
BFA	1.016	1.015	1.15	0.73	0.20	0.24	0.760	1.14
BGD	1.076	1.072	1.15	0.64	0.13	0.24	0.858	0.37
BGR	1.107	1.109	1.15	0.56	0.14	0.26	0.630	0.52
BHR	1.051	1.051	1.15	0.68	0.13	0.25	0.790	0.54
BHS	1.113	1.116	1.15	0.51	0.11	0.19	0.532	1.45
BIH	1.098	1.097	1.15	0.59	0.16	0.24	0.696	0.48
BLZ	1.118	1.120	1.15	0.52	0.17	0.25	0.556	0.90
BMU	1.140	1.130	1.15	0.40	0.05	0.13	0.572	0.78
BOL	1.146	1.144	1.15	0.43	0.28	0.42	0.767	0.17
BRA	1.095	1.092	1.15	0.61	0.17	0.33	0.910	0.16
BRB	1.094	1.098	1.15	0.59	0.16	0.24	0.631	0.95
BRN	1.135	1.129	1.15	0.46	0.15	0.25	0.755	0.19
BTN	1.144	1.142	1.15	0.40	0.14	0.26	0.665	0.25

BWA	1.071	1.071	1.15	0.66	0.23	0.26	0.535	1.49
CAF	1.115	1.116	1.15	0.55	0.19	0.23	0.838	0.28
CAN	1.111	1.111	1.15	0.54	0.13	0.26	0.663	0.43
CHE	1.152	1.146	1.15	0.31	0.07	0.29	0.669	0.14
CHL	1.127	1.126	1.15	0.50	0.18	0.29	0.721	0.27
CHN	1.102	1.103	1.15	0.65	0.39	0.54	0.905	0.18
CIV	1.102	1.101	1.15	0.59	0.17	0.23	0.809	0.33
CMR	1.128	1.128	1.15	0.48	0.13	0.21	0.797	0.26
COD	1.106	1.102	1.15	0.57	0.17	0.23	0.780	0.40
COG	1.137	1.127	1.15	0.44	0.14	0.24	0.766	0.26
COL	1.124	1.121	1.15	0.48	0.11	0.36	0.766	0.27
CPV	1.083	1.080	1.15	0.63	0.19	0.24	0.677	0.89
CRI	1.105	1.103	1.15	0.56	0.13	0.25	0.690	0.45
CUB	1.133	1.131	1.15	0.47	0.15	0.20	0.703	0.33
CYP	1.106	1.104	1.15	0.56	0.16	0.24	0.545	0.76
CZE	1.101	1.103	1.15	0.57	0.14	0.34	0.537	0.81
DEU	1.105	1.104	1.15	0.54	0.07	0.27	0.629	0.50
DJI	1.104	1.102	1.15	0.57	0.15	0.21	0.719	0.68
DNK	1.113	1.111	1.15	0.52	0.10	0.21	0.486	0.77
DOM	1.123	1.118	1.15	0.51	0.14	0.22	0.740	0.28
DZA	1.151	1.147	1.15	0.38	0.15	0.23	0.646	0.31
ECU	1.083	1.083	1.15	0.68	0.32	0.45	0.846	0.38
EGY	1.116	1.117	1.15	0.54	0.17	0.21	0.796	0.27
ERI	1.115	1.116	1.15	0.55	0.15	0.19	0.874	0.15
ESP	1.112	1.110	1.15	0.55	0.17	0.28	0.636	0.49
EST	1.097	1.100	1.15	0.58	0.18	0.30	0.368	1.69
FIN	1.102	1.099	1.15	0.57	0.15	0.29	0.571	0.71
FJI	1.098	1.100	1.15	0.59	0.18	0.26	0.606	0.98
FRA	1.118	1.116	1.15	0.51	0.11	0.24	0.615	0.44

GAB	1.139	1.135	1.15	0.43	0.13	0.22	0.794	0.20
GBR	1.141	1.135	1.15	0.42	0.12	0.22	0.513	0.43
GEO	1.120	1.119	1.15	0.55	0.26	0.40	0.659	0.43
GHA	1.155	1.149	1.15	0.33	0.11	0.20	0.741	0.15
GIN	1.148	1.143	1.15	0.38	0.14	0.23	0.673	0.29
GMB	1.091	1.087	1.15	0.62	0.21	0.26	0.783	0.60
GRC	1.119	1.116	1.15	0.54	0.20	0.26	0.576	0.58
GTM	1.126	1.124	1.15	0.49	0.14	0.24	0.704	0.29
HKG	1.097	1.117	1.15	0.58	0.31	0.51	0.169	4.67
HND	1.056	1.056	1.15	0.68	0.17	0.28	0.756	0.62
HRV	1.107	1.102	1.15	0.55	0.13	0.25	0.607	0.59
HTI	1.124	1.119	1.15	0.50	0.14	0.21	0.801	0.19
HUN	1.065	1.069	1.15	0.65	0.18	0.36	0.384	2.39
IDN	1.126	1.125	1.15	0.55	0.33	0.47	0.879	0.13
IND	1.127	1.121	1.15	0.53	0.27	0.38	0.872	0.16
IRL	1.143	1.136	1.15	0.41	0.14	0.30	0.377	0.68
IRN	1.119	1.113	1.15	0.55	0.23	0.37	0.867	0.24
ISL	1.128	1.125	1.15	0.48	0.13	0.25	0.572	0.45
ISR	1.104	1.102	1.15	0.57	0.17	0.25	0.636	0.58
ITA	1.125	1.123	1.15	0.50	0.15	0.28	0.661	0.34
JAM	1.119	1.115	1.15	0.52	0.14	0.22	0.649	0.49
JOR	1.064	1.064	1.15	0.66	0.17	0.28	0.639	1.07
JPN	1.115	1.113	1.15	0.54	0.13	0.25	0.837	0.17
KAZ	1.085	1.083	1.15	0.66	0.27	0.52	0.828	0.37
KEN	1.121	1.115	1.15	0.53	0.21	0.37	0.785	0.30
KGZ	1.099	1.095	1.15	0.63	0.33	0.65	0.760	0.45
KHM	1.105	1.102	1.15	0.57	0.17	0.27	0.666	0.65
KOR	1.026	1.024	1.15	0.74	0.20	0.49	0.778	0.94
LAO	1.144	1.136	1.15	0.41	0.15	0.23	0.781	0.14

LBN	1.107	1.109	1.15	0.55	0.15	0.24	0.560	0.81
LBY	1.143	1.135	1.15	0.40	0.11	0.21	0.855	0.07
LKA	1.145	1.141	1.15	0.38	0.10	0.21	0.660	0.27
LSO	1.111	1.111	1.15	0.57	0.21	0.29	0.680	0.55
LTU	1.115	1.110	1.15	0.53	0.19	0.38	0.456	0.91
LUX	1.078	1.098	1.15	0.57	0.07	0.18	0.184	4.54
LVA	1.121	1.118	1.15	0.51	0.17	0.29	0.469	0.78
MAC	1.137	1.137	1.15	0.42	0.09	0.17	0.439	0.57
MAR	1.119	1.113	1.15	0.52	0.14	0.24	0.728	0.36
MDG	1.120	1.118	1.15	0.52	0.16	0.24	0.696	0.45
MDV	1.082	1.090	1.15	0.62	0.18	0.25	0.612	1.59
MEX	1.111	1.109	1.15	0.55	0.16	0.29	0.615	0.56
MKD	1.105	1.106	1.15	0.60	0.26	0.34	0.617	0.69
MLI	1.111	1.110	1.15	0.56	0.16	0.21	0.801	0.31
MLT	1.013	1.016	1.15	0.73	0.14	0.35	0.485	2.91
MMR	0.854	0.853	1.15	0.82	0.13	0.32	0.998	0.02
MNE	1.132	1.132	1.15	0.50	0.21	0.23	0.713	0.27
MNG	1.122	1.121	1.15	0.52	0.19	0.28	0.633	0.50
MOZ	1.138	1.136	1.15	0.44	0.15	0.23	0.718	0.24
MRT	1.067	1.067	1.15	0.66	0.19	0.30	0.697	0.98
MUS	1.088	1.086	1.15	0.61	0.20	0.34	0.532	1.08
MWI	1.108	1.105	1.15	0.56	0.17	0.24	0.683	0.56
MYS	1.097	1.099	1.15	0.60	0.25	0.56	0.579	0.92
NAM	1.092	1.092	1.15	0.60	0.17	0.26	0.568	0.89
NER	1.071	1.074	1.15	0.66	0.22	0.21	0.788	0.62
NGA	1.078	1.075	1.15	0.66	0.25	0.18	0.721	0.81
NIC	1.112	1.107	1.15	0.56	0.20	0.26	0.694	0.43
NLD	1.118	1.116	1.15	0.51	0.13	0.25	0.383	1.10
NOR	1.138	1.137	1.15	0.42	0.10	0.23	0.573	0.35

NPL	1.122	1.115	1.15	0.50	0.12	0.22	0.713	0.40
NZL	1.132	1.130	1.15	0.48	0.18	0.29	0.739	0.22
OMN	1.131	1.126	1.15	0.47	0.15	0.25	0.672	0.31
PAK	1.128	1.121	1.15	0.48	0.12	0.20	0.894	0.11
PAN	1.102	1.103	1.15	0.57	0.15	0.24	0.637	0.72
PER	1.116	1.113	1.15	0.57	0.24	0.31	0.858	0.17
PHL	1.115	1.112	1.15	0.55	0.22	0.44	0.707	0.39
PNG	1.107	1.103	1.15	0.58	0.20	0.24	0.701	0.47
POL	1.131	1.129	1.15	0.48	0.18	0.32	0.653	0.32
PRT	1.097	1.098	1.15	0.58	0.14	0.29	0.496	0.99
PRY	1.100	1.099	1.15	0.65	0.40	0.57	0.841	0.31
PYF	1.094	1.097	1.15	0.60	0.16	0.23	0.689	0.58
QAT	1.161	1.153	1.15	0.28	0.12	0.19	0.779	0.07
ROU	1.135	1.134	1.15	0.48	0.25	0.44	0.692	0.27
RUS	1.128	1.126	1.15	0.51	0.22	0.35	0.843	0.17
RWA	1.112	1.112	1.15	0.56	0.16	0.21	0.815	0.25
SAU	1.145	1.142	1.15	0.42	0.18	0.24	0.617	0.31
SEN	1.134	1.131	1.15	0.49	0.21	0.23	0.742	0.31
SGP	1.003	1.020	1.15	0.76	0.21	0.33	0.272	8.28
SLE	1.065	1.067	1.15	0.64	0.10	0.15	0.775	0.63
SLV	1.100	1.101	1.15	0.58	0.15	0.25	0.617	0.65
SMR	1.140	1.134	1.15	0.45	0.18	0.25	0.741	0.18
SRB	1.104	1.104	1.15	0.59	0.18	0.24	0.913	0.11
SUR	1.085	1.087	1.15	0.63	0.18	0.30	0.753	0.58
SVK	1.054	1.060	1.15	0.67	0.18	0.33	0.405	2.53
SVN	1.113	1.116	1.15	0.52	0.16	0.34	0.371	1.25
SWE	1.137	1.134	1.15	0.43	0.12	0.26	0.464	0.55
SWZ	1.055	1.059	1.15	0.70	0.28	0.31	0.490	2.32
SYC	1.118	1.114	1.15	0.55	0.24	0.28	0.590	1.02

SYR	1.107	1.103	1.15	0.55	0.10	0.21	0.844	0.23
TCD	1.126	1.120	1.15	0.51	0.16	0.20	0.918	0.10
TGO	1.107	1.109	1.15	0.57	0.19	0.24	0.711	0.60
THA	1.070	1.071	1.15	0.68	0.28	0.44	0.723	0.82
TKM	1.091	1.091	1.15	0.61	0.21	0.29	0.565	1.01
TTO	1.139	1.136	1.15	0.43	0.14	0.30	0.747	0.19
TUN	1.078	1.080	1.15	0.64	0.18	0.29	0.635	0.96
TUR	1.120	1.115	1.15	0.52	0.18	0.35	0.680	0.39
TWN	1.103	1.105	1.15	0.57	0.15	0.36	0.717	0.41
TZA	1.077	1.080	1.15	0.64	0.21	0.28	0.511	1.84
UGA	1.103	1.102	1.15	0.58	0.16	0.22	0.741	0.47
UKR	1.088	1.089	1.15	0.63	0.21	0.44	0.712	0.57
URY	1.106	1.105	1.15	0.57	0.17	0.26	0.704	0.48
USA	1.119	1.120	1.15	0.52	0.11	0.20	0.751	0.25
UZB	1.097	1.096	1.15	0.63	0.24	0.48	0.944	0.10
VEN	1.125	1.125	1.15	0.54	0.26	0.37	0.906	0.12
VNM	1.114	1.111	1.15	0.57	0.32	0.58	0.732	0.43
VUT	1.098	1.095	1.15	0.60	0.21	0.26	0.721	0.75
YEM	1.095	1.094	1.15	0.59	0.12	0.23	0.819	0.29
ZAF	1.125	1.119	1.15	0.49	0.12	0.32	0.752	0.24
ZMB	1.120	1.117	1.15	0.51	0.14	0.23	0.709	0.34
ZWE	1.130	1.129	1.15	0.49	0.16	0.23	0.813	0.22

Appendix References

Melitz, Marc J., and Stephen J. Redding. 2014. Heterogeneous Firms and Trade. In *Handbook of International Economics*, volume 4, edited by Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff. Amsterdam: North-Holland, pp. 1–54.